Active Filters - Revisited

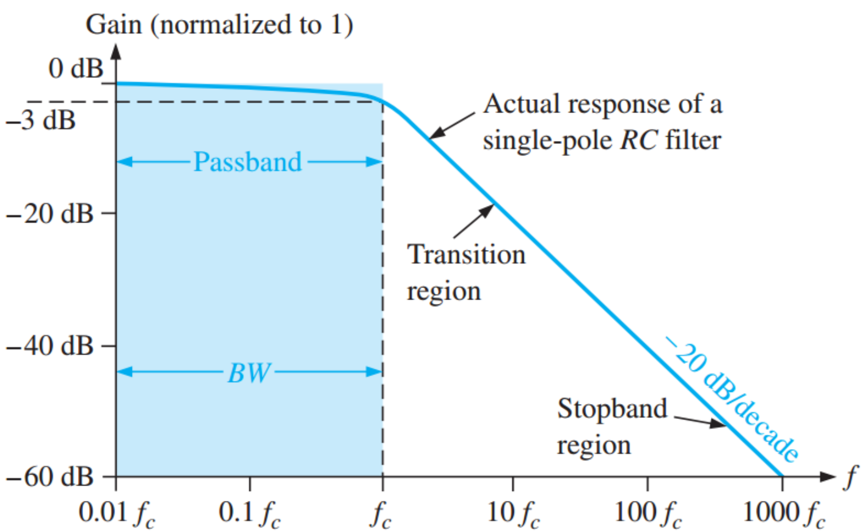
Sources:

Electronic Devices
by
Thomas L. Floyd.

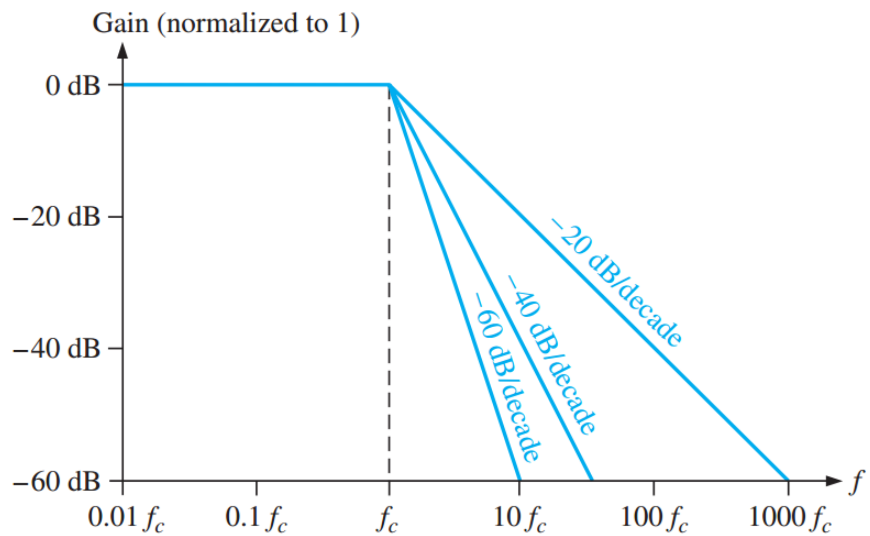
&

Electronic Devices and Circuit Theory
by
Robert L. Boylestad, Louis Nashelsky

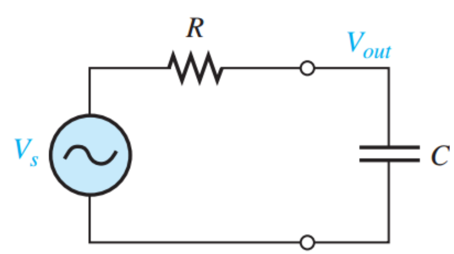
Ideal and Practical Filters



Comparison of an ideal low-pass filter response (blue area) with actual response. Although not shown on log scale, response extends down to $f_c = 0$.



Idealized low-pass filter responses

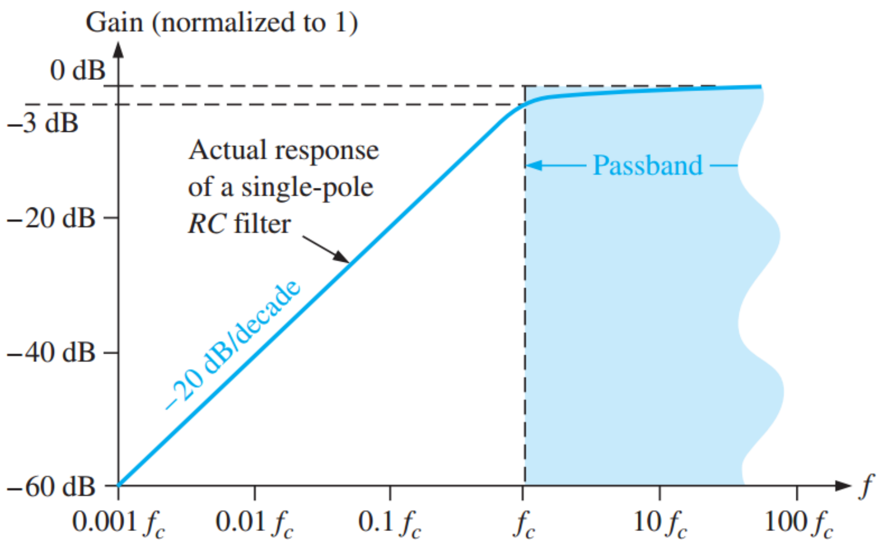


Basic low-pass circuit

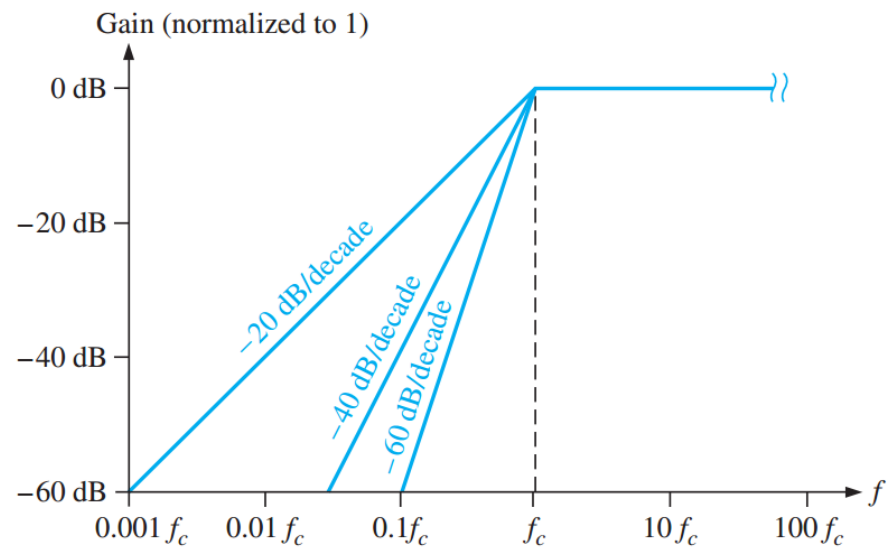
$$BW = f_c$$

$$f_c = \frac{1}{2\pi RC}$$

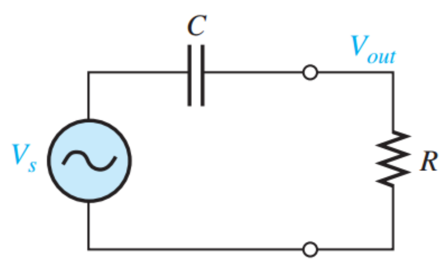
Ideal and Practical Filters



Comparison of an ideal high-pass filter response (blue area) with actual response



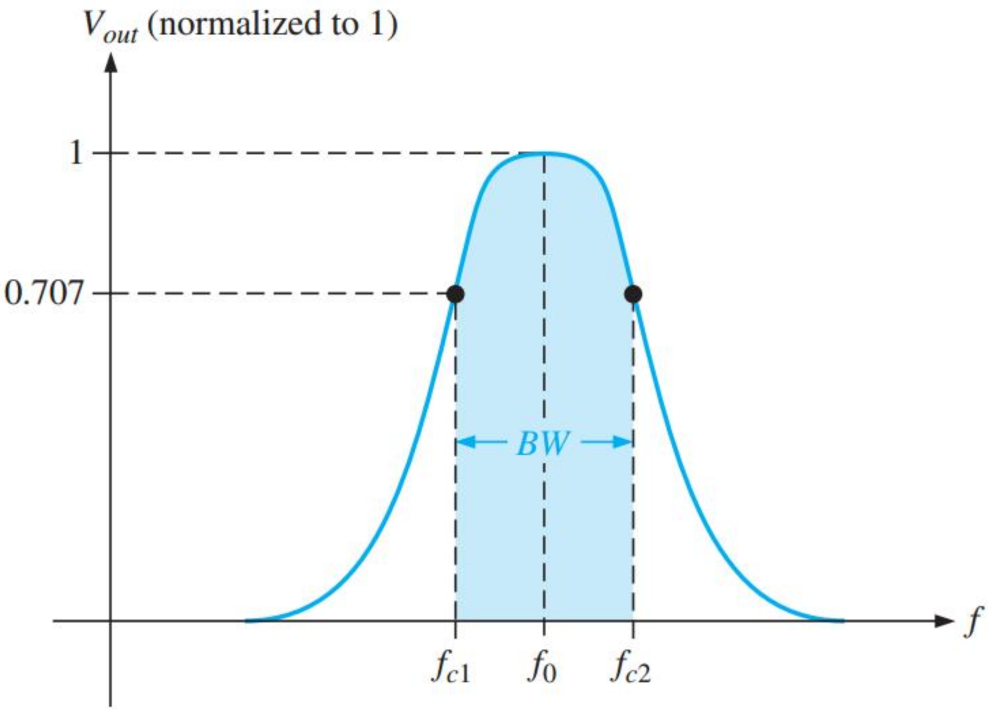
Idealized high-pass filter responses



Basic high-pass circuit

$$f_c = \frac{1}{2\pi RC}$$

Ideal and Practical Filters



$$BW = f_{c2} - f_{c1}$$

$$f_0 = \sqrt{f_{c1} f_{c2}}$$

Quality Factor (Q) of a band-pass filter is the ratio of the center frequency to the bandwidth.

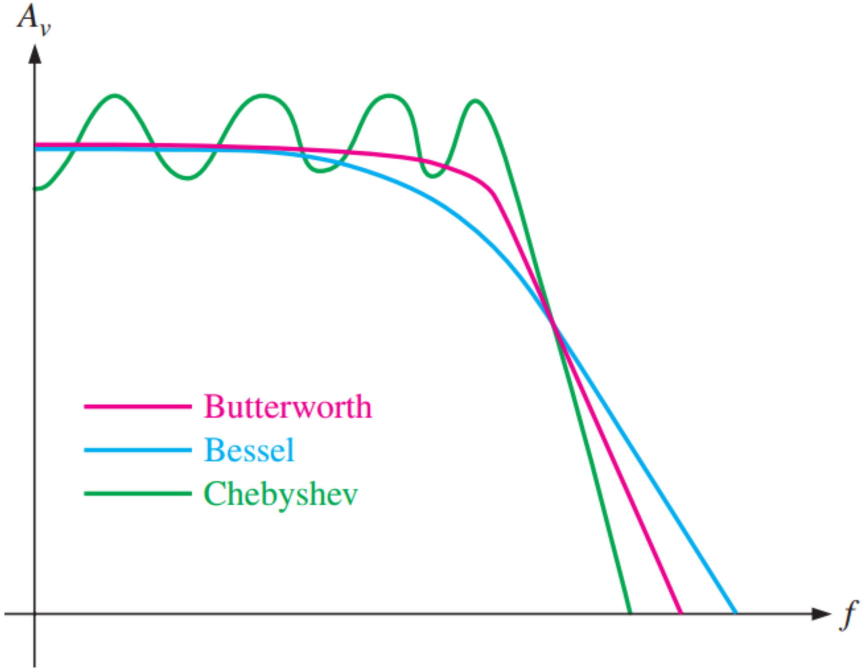
$$Q = \frac{f_0}{BW}$$

The quality factor (Q) can also be expressed in terms of the damping factor (DF) of the filter as

$$Q = \frac{1}{DF}$$

Ideal and Practical Filters

Butterworth, Chebyshev, or Bessel response characteristics can be realized with most active filter circuit configurations by proper selection of certain component values.

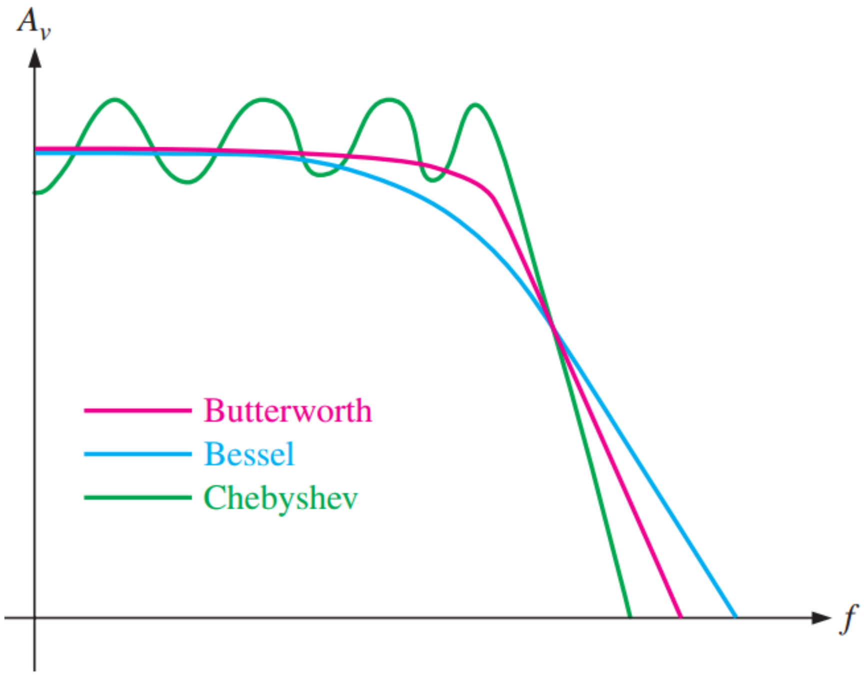


The Butterworth Characteristic

- Provides a very flat amplitude response in the passband and a roll-off rate of -20 dB/decade/pole
- Phase response is not linear
- A pulse will cause overshoots on the output because each frequency component of the pulse's rising and falling edges experiences a different time delay
- Normally used when all frequencies in the passband must have the same gain.
- Often referred to as a maximally flat response.

Ideal and Practical Filters

Butterworth, Chebyshev, or Bessel response characteristics can be realized with most active filter circuit configurations by proper selection of certain component values.

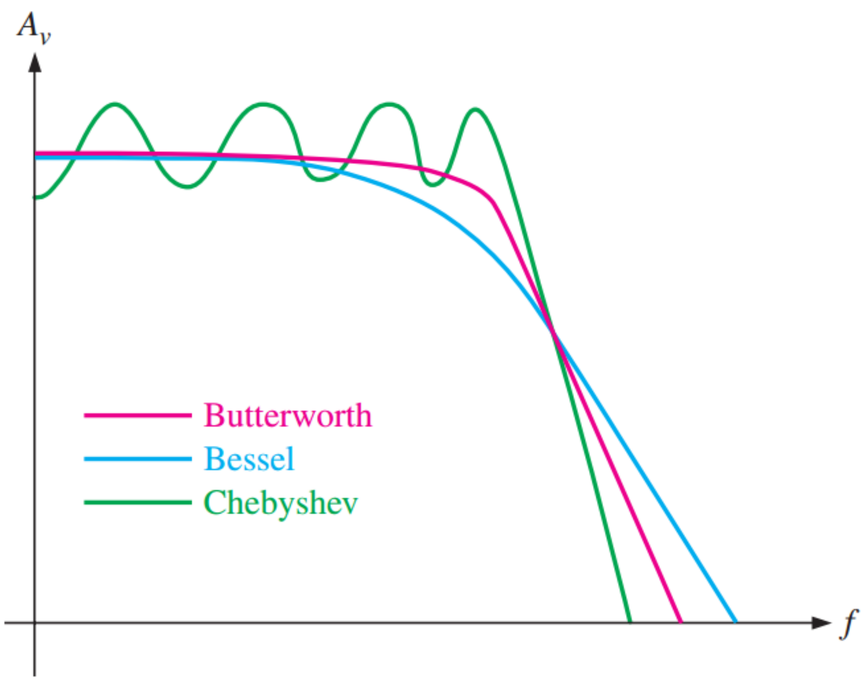


The Chebyshev Characteristic

- Useful when a rapid roll-off is required
- Because it provides a roll-off rate greater than 20 dB/decade/pole
- Filters can be implemented with fewer poles and less complex circuitry for a given roll-off rate
- Characterized by overshoot or ripples in the passband (depending on the number of poles)
- and an even less linear phase response than the Butterworth.

Ideal and Practical Filters

Butterworth, Chebyshev, or Bessel response characteristics can be realized with most active filter circuit configurations by proper selection of certain component values.

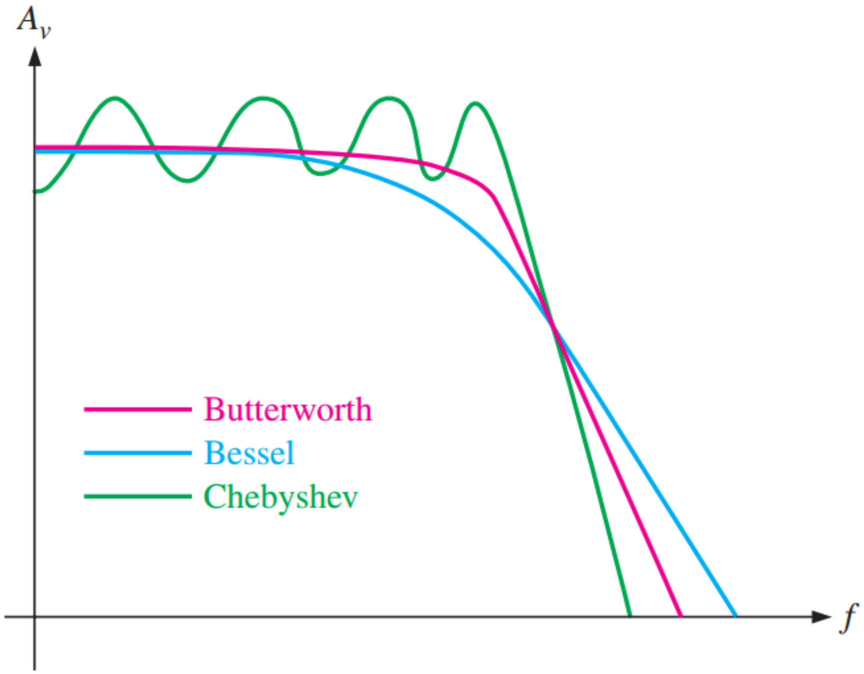


The Bessel Characteristic

- Response exhibits a linear phase characteristic
- Meaning that the phase shift increases linearly with frequency
- Result is almost no overshoot on the output with a pulse input
- For this reason, filters with the Bessel response are used for filtering pulse waveforms without distorting the shape of the waveform.

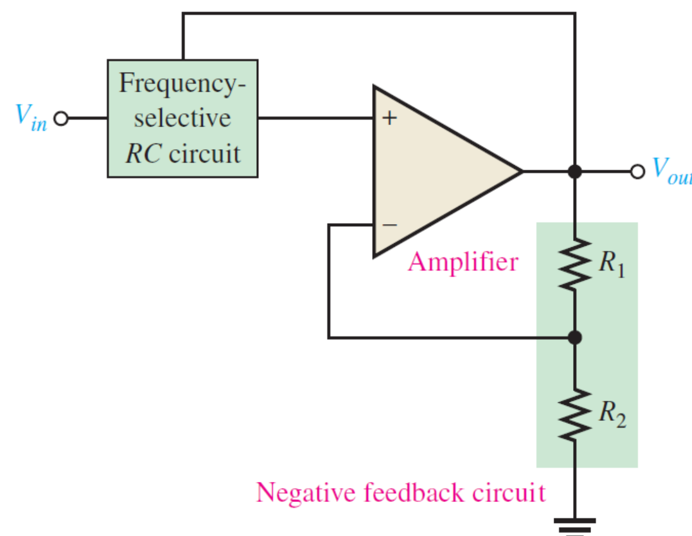
Ideal and Practical Filters

- An active filter can be designed to have either a Butterworth, Chebyshev, or Bessel response characteristic regardless of whether it is a low-pass, high-pass, band-pass,
- The damping factor (DF) of an active filter circuit determines which response characteristic the filter exhibits

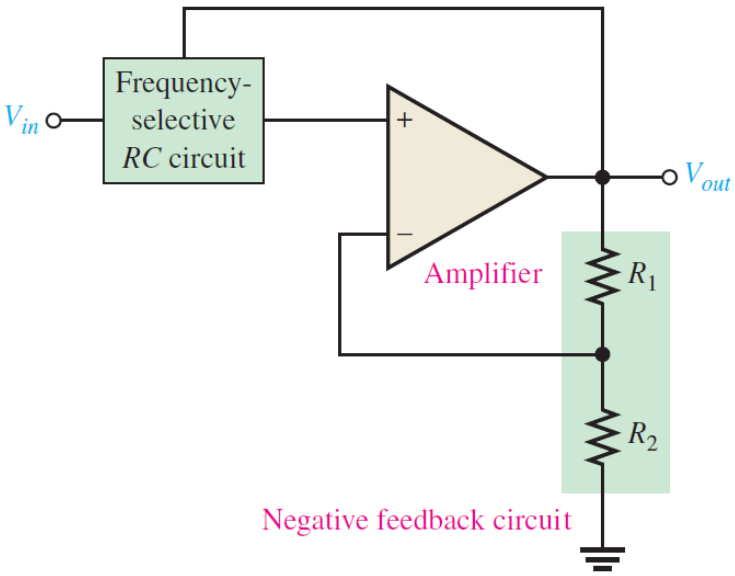


- A generalized active filter is shown in figure below
- Includes an amplifier, a negative feedback circuit, and a filter section
- Damping factor determined by negative feedback circuit is given by

$$DF = 2 - \frac{R_1}{R_2}$$

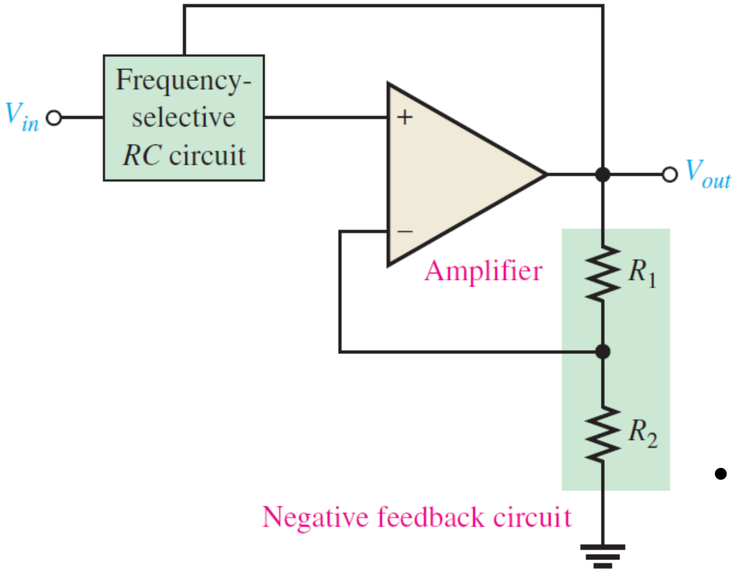


Ideal and Practical Filters



- Damping factor affects filter response by negative feedback action
- Any attempted increase or decrease in the output voltage is offset by the opposing effect of the negative feedback
- This tends to make the response curve flat in the passband of the filter if the value for the damping factor is precisely set
- By advanced mathematics, which we will not cover, values for the damping factor have been derived for various orders of filters to achieve the maximally flat response of the Butterworth characteristic
- The value of the damping factor required to produce a desired response characteristic depends on the order (number of poles) of the filter
- A pole, for our purposes, is simply a circuit with one resistor and one capacitor. The more poles a filter has, the faster its roll-off rate is
- To achieve a second-order Butterworth response, for example, the damping factor must be 1.414.

Ideal and Practical Filters



- To achieve a second-order Butterworth response, for example, the damping factor must be 1.414
- To implement this damping factor, the feedback resistor ratio must be

$$\frac{R_1}{R_2} = 2 - DF = 2 - 1.414 = 0.586$$

- This ratio gives the closed-loop gain of the noninverting amplifier portion of the filter, derived as follows

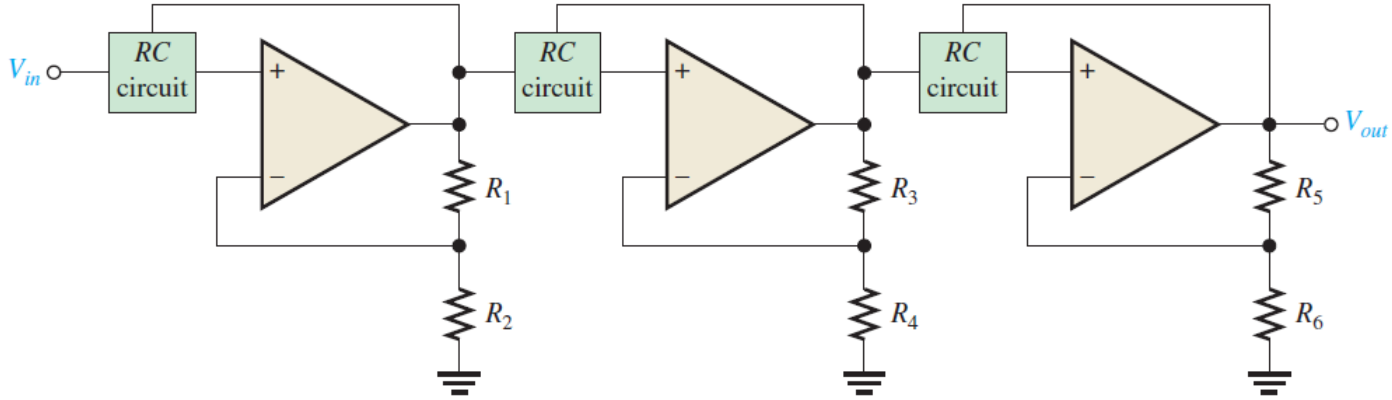
$$A_{cl(\mathrm{NI})} = \frac{1}{B} = \frac{1}{R_2/(R_1 + R_2)} = \frac{R_1 + R_2}{R_2} = \frac{R_1}{R_2} + 1 = 0.586 + 1 = 1.586$$

Ideal and Practical Filters

- To produce a filter that has a steeper transition region it is necessary to add additional circuitry to the basic filter.
- Responses that are steeper than in the transition region cannot be obtained by simply cascading identical RC stages (due to loading effects)
- However, by combining an op-amp with frequency-selective feedback circuits, filters can be designed with roll-off rates of or more dB/decade
- Filters that include one or more op-amps in the design are called active filters
- These filters can optimize the roll-off rate or other attribute (such as phase response) with a particular filter design
- In general, the more poles the filter uses, the steeper its transition region will be
- The exact response depends on the type of filter and the number of poles

Ideal and Practical Filters

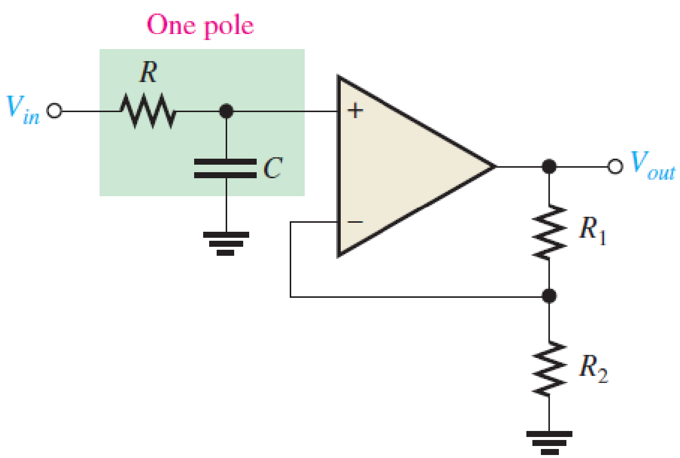
- The number of poles determines the roll-off rate of the filter
- A Butterworth response produces -20 dB/decade/pole
  - a first-order (one-pole) filter has a roll-off of -20 dB/decade
  - a second-order (two-pole) filter has a roll-off rate of -40 dB/decade
  - a third-order (three-pole) filter has a roll-off rate of -60 dB/decade …
- Generally, to obtain a filter with three poles or more, one-pole or two-pole filters are cascaded, as shown in figure below
- To obtain a third-order filter, for example, cascade a second-order and a first-order filter
- To obtain a fourth-order filter, cascade two second-order filters; and so on ,
- Each filter in a cascaded arrangement is called a stage or section.

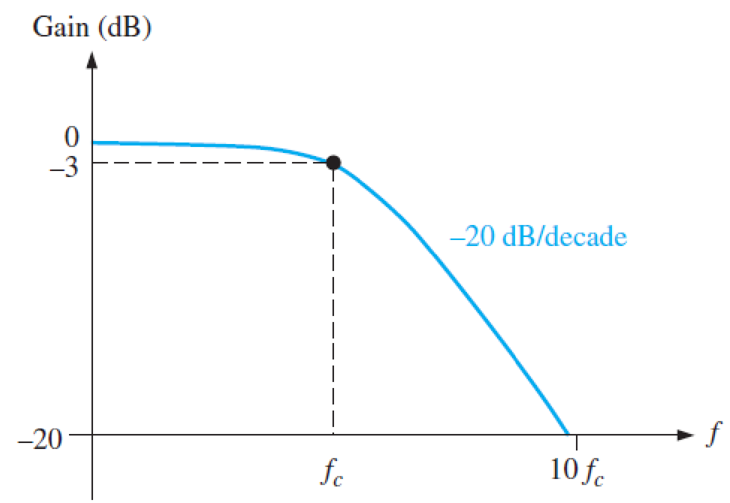


Active Filters – Low-Pass Filters

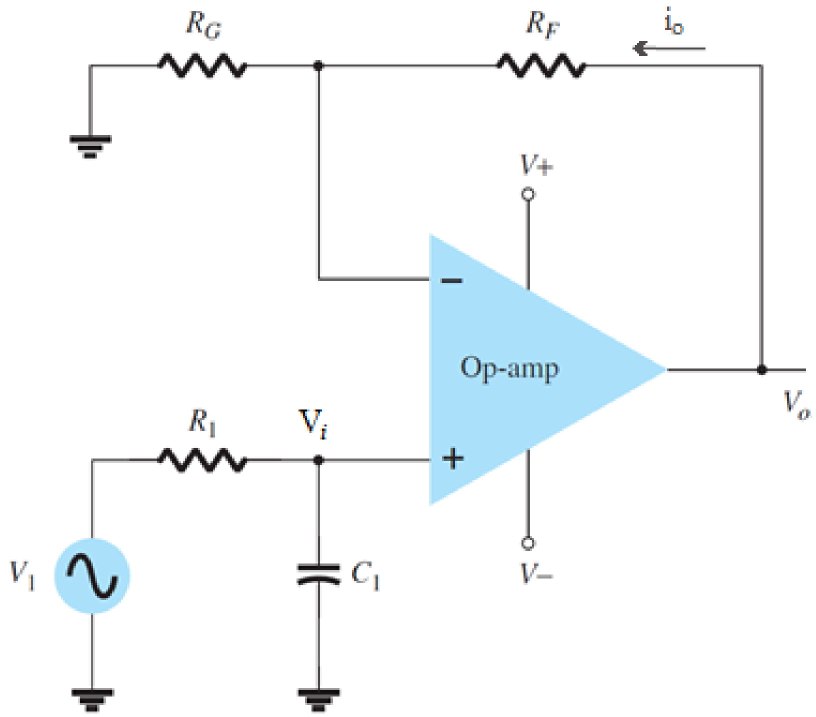
A Single Pole Low-Pass Filter

$$A_{cl(NI)} = \frac{R_1}{R_2} + 1 \qquad f_c = \frac{1}{2\pi RC}$$





Active Filters – Low-Pass Filters



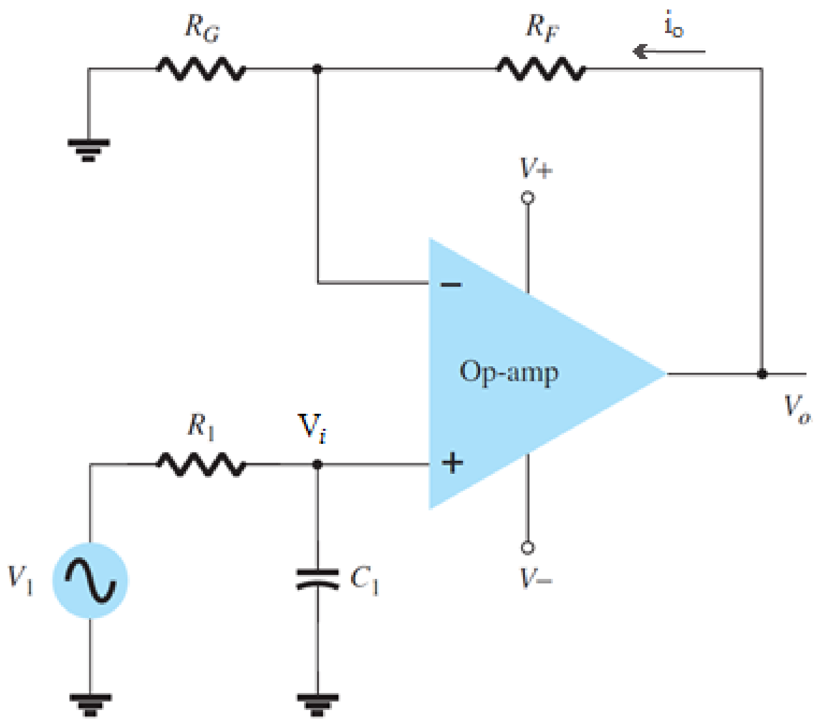
$$V_i = \frac{V_1}{R_1 + \frac{1}{j\omega C_1}} \cdot \frac{1}{j\,\omega C_1} \Rightarrow V_i = \frac{V_1}{1 + j\omega R_1 C_1}$$

$$i_o = \frac{V_o - V_i}{R_F} \approx \frac{V_i}{R_G} \Rightarrow V_o = \left(1 + \frac{R_F}{R_G}\right) V_i$$

Combining:

$$V_o = \left(1 + \frac{R_F}{R_G}\right)\left(\frac{1}{1 + j\omega R_1 C_1}\right) V_1$$

Active Filters – Low-Pass Filters



$$\frac{V_o}{V_1} = A_v = \left(1 + \frac{R_F}{R_G}\right)\left(\frac{1}{1 + j\omega R_1 C_1}\right)$$

Real number (gain)

>1 for a low-pass filter with voltage gain

Low-pass filter

$$f_{OL} = \frac{1}{2\,\pi R_1 C_1}$$

cut-off frequency

Active Filters – Low-Pass Filters

$$\frac{V_o}{V_1} = A_v = \left(1 + \frac{R_F}{R_G}\right)\left(\frac{1}{1 + j\omega R_1 C_1}\right)$$

Real number (gain)

Low-pass filter

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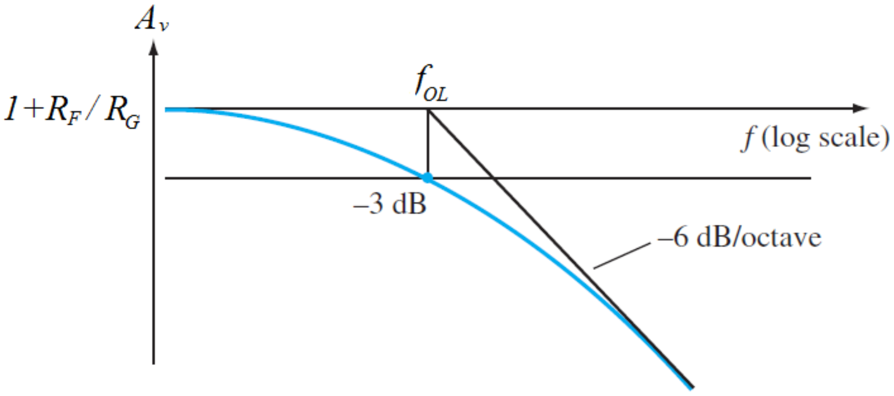
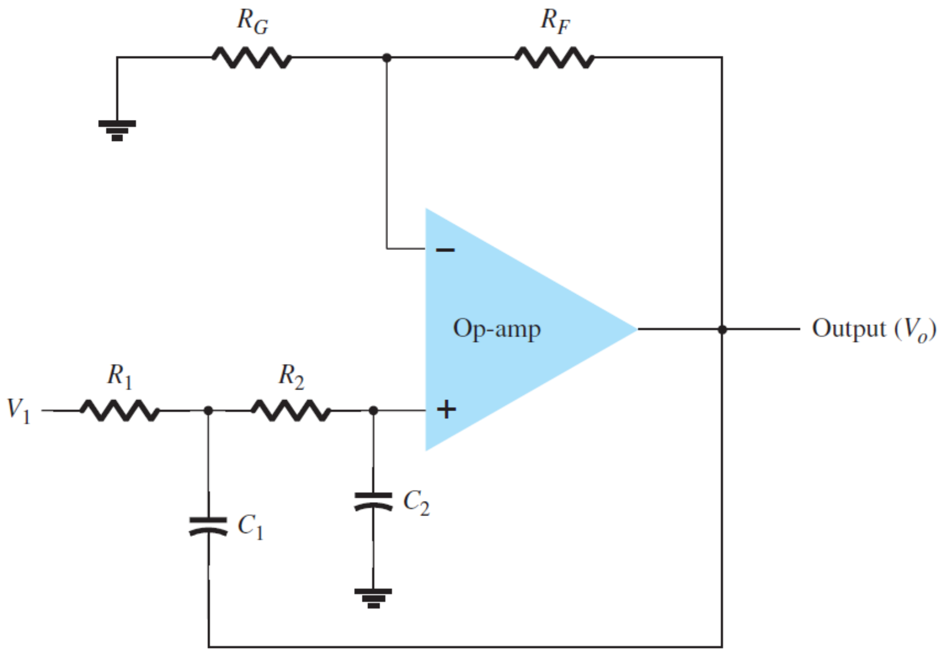
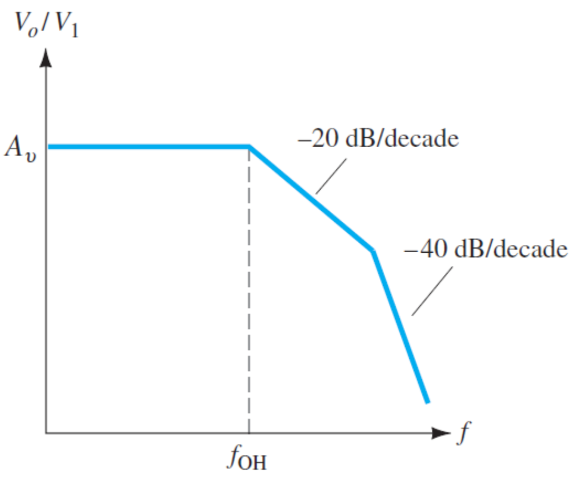
$$f_{OL} = \frac{1}{2\,\pi R_1 C_1}$$

cut-off frequency



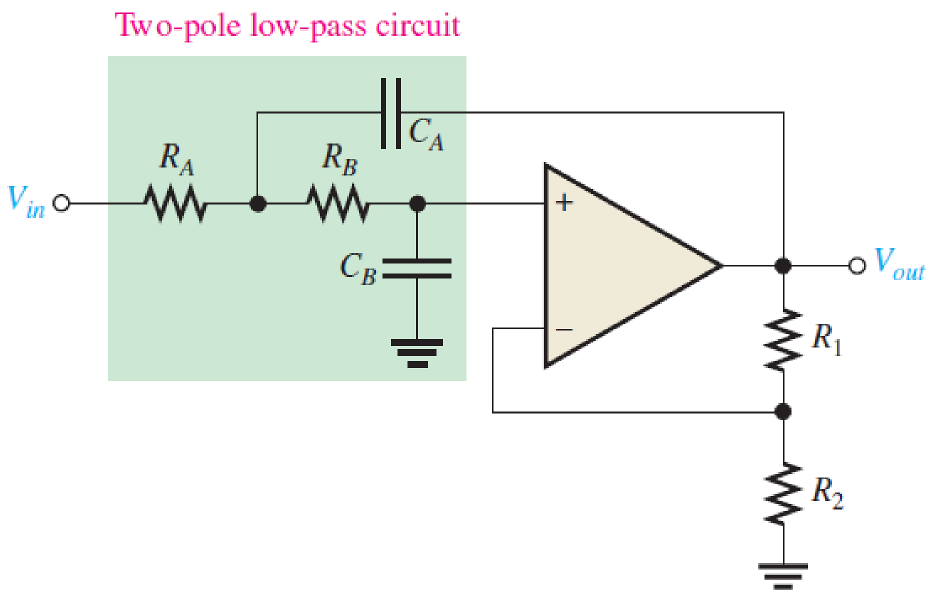
Active Filters – Low-Pass Filters





Active Filters – Low-Pass Filters

The Sallen-Key Low-Pass Filter



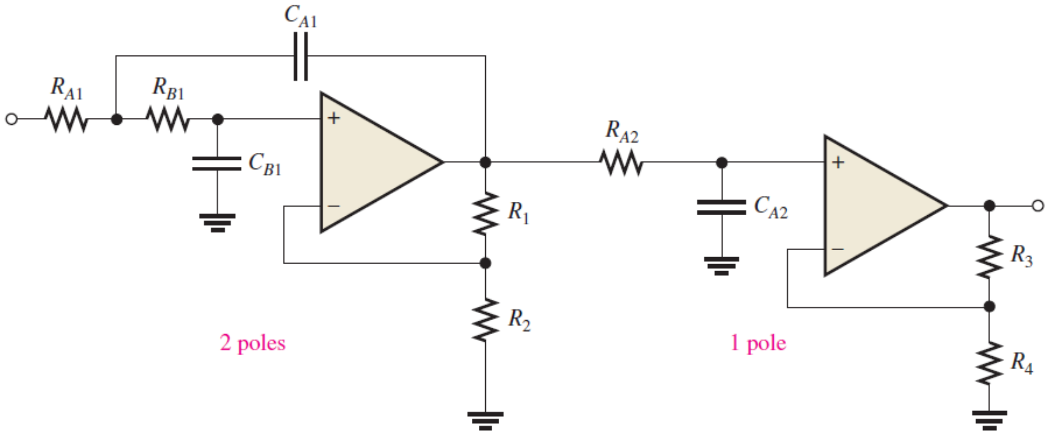
- There are two low-pass *RC* circuits that provide a roll-off of -40 dB/decade above the critical frequency (assuming a Butterworth characteristic)
- One *RC* circuit consists of R_A and C_A and the second circuit consists of R_B and C_B
- A unique feature is the capacitor that provides feedback for shaping the response near the edge of passband
- If $R_A = R_B = R$ and $C_A = C_B = C$

$$f_c = \frac{1}{2\pi\sqrt{R_A R_B C_A C_B}}$$

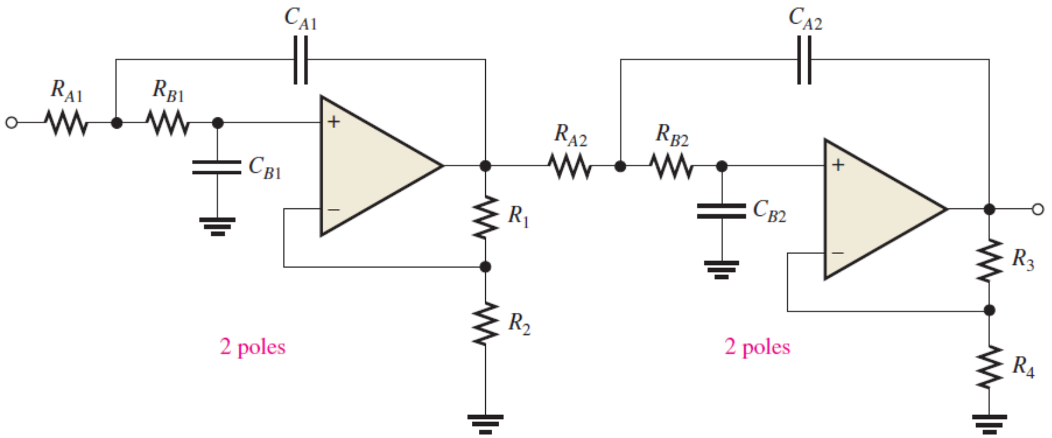
$$f_c = \frac{1}{2\pi RC}$$

Active Filters – Low-Pass Filters

Cascaded Low-Pass Filter



Third-order configuration

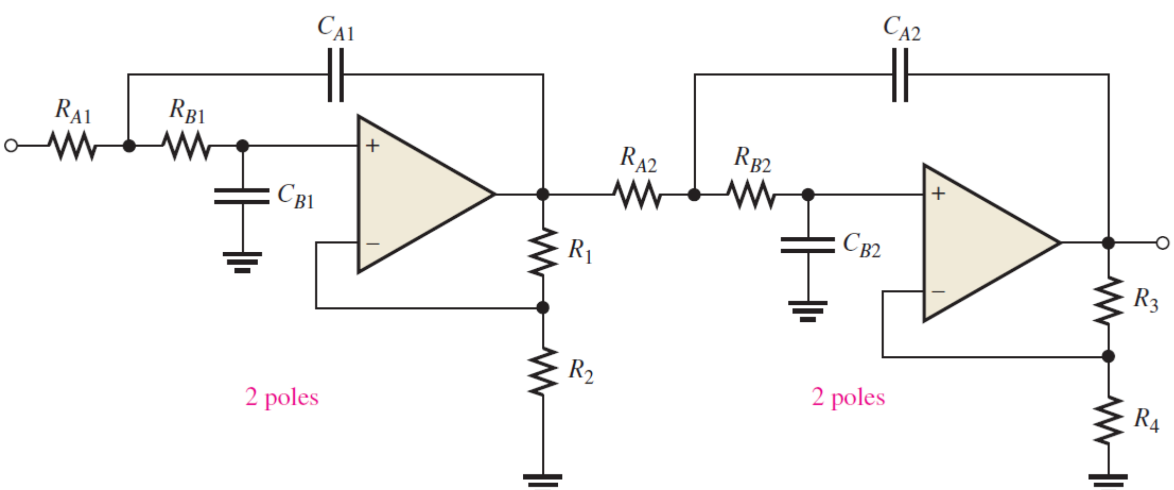


Fourth-order configuration

Ideal and Practical Filters

Values for the Butterworth response

ORDER	ROLL-OFF DB/DECADE	1ST STAGE POLES	1ST STAGE DF	1ST STAGE R_1/R_2	2ND STAGE POLES	2ND STAGE DF	2ND STAGE R_3/R_4	3RD STAGE POLES	3RD STAGE DF	3RD STAGE R_5/R_6
1	−20	1	Optional							
2	−40	2	1.414	0.586						
3	−60	2	1.00	1	1	1.00	1			
4	−80	2	1.848	0.152	2	0.765	1.235			
5	−100	2	1.00	1	2	1.618	0.382	1	0.618	1.382
6	−120	2	1.932	0.068	2	1.414	0.586	2	0.518	1.482



- Determine the capacitance values for a critical frequency of 2680 Hz if all the resistors in the *RC* low-pass circuits are 1.8 KΩ.
- Also select values for the feedback resistors to get a Butterworth response

Ideal and Practical Filters

ORDER	ROLL-OFF DB/DECADE	1ST STAGE			2ND STAGE			3RD STAGE		
		POLES	*DF*	R_1/R_2	POLES	*DF*	R_3/R_4	POLES	*DF*	R_5/R_6
1	−20	1	Optional							
2	−40	2	1.414	0.586						
3	−60	2	1.00	1	1	1.00	1			
4	−80	2	1.848	0.152	2	0.765	1.235			
5	−100	2	1.00	1	2	1.618	0.382	1	0.618	1.382
6	−120	2	1.932	0.068	2	1.414	0.586	2	0.518	1.482

- Determine the capacitance values for a critical frequency of 2680 Hz if all the resistors in the *RC* low-pass circuits are 1.8 KΩ.
- Also select values for the feedback resistors to get a Butterworth response

Both stages must have the same f_c. Assuming equal-value capacitors,

$$f_c = \frac{1}{2\pi RC}$$

$$C = \frac{1}{2\pi R f_c} = \frac{1}{2\pi(1.8\,\text{k}\Omega)(2680\,\text{Hz})} = 0.033\,\mu\text{F}$$

$$C_{A1} = C_{B1} = C_{A2} = C_{B2} = \mathbf{0.033\,\mu F}$$

Also select $R_2 = R_4 = 1.8\,\text{k}\Omega$ for simplicity. Refer to Table 15–1. For a Butterworth response in the first stage, $DF = 1.848$ and $R_1/R_2 = 0.152$. Therefore,

$$R_1 = 0.152R_2 = 0.152(1800\,\Omega) = \mathbf{274\,\Omega}$$

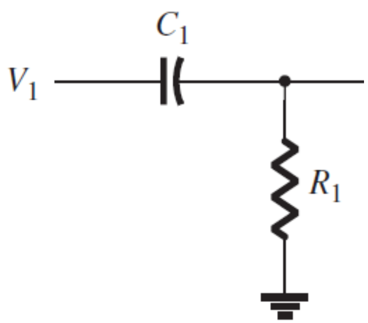
Choose $R_1 = 270\,\Omega$.

In the second stage, $DF = 0.765$ and $R_3/R_4 = 1.235$. Therefore,

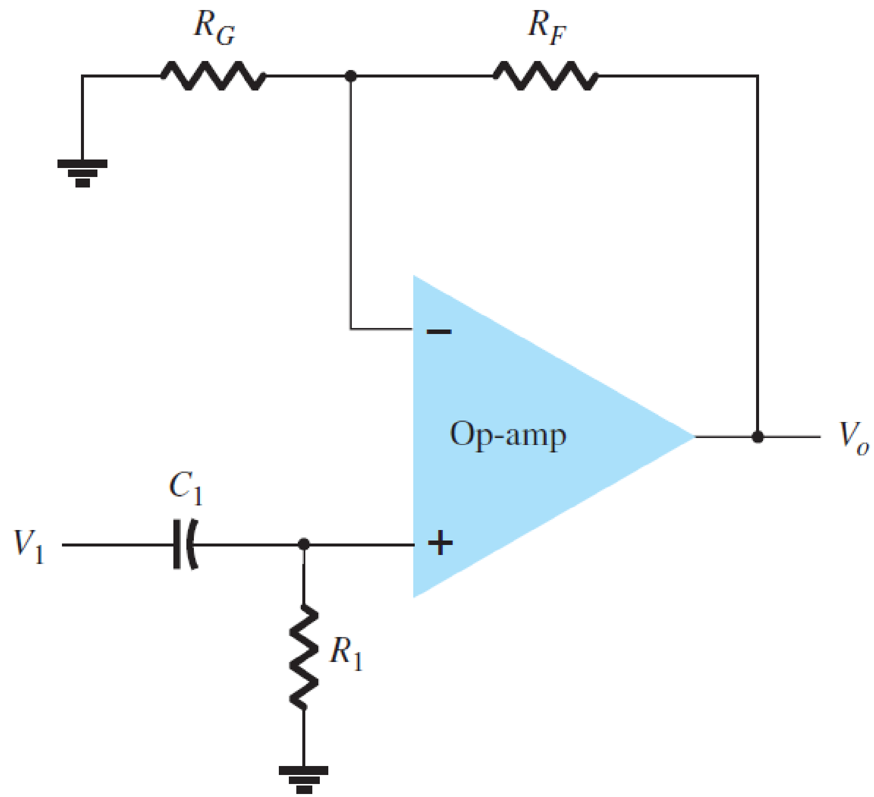
$$R_3 = 1.235R_4 = 1.235(1800\,\Omega) = \mathbf{2.22\,k\Omega}$$

Choose $R_3 = 2.2\,\text{k}\Omega$.

Active Filters – High-Pass Filters



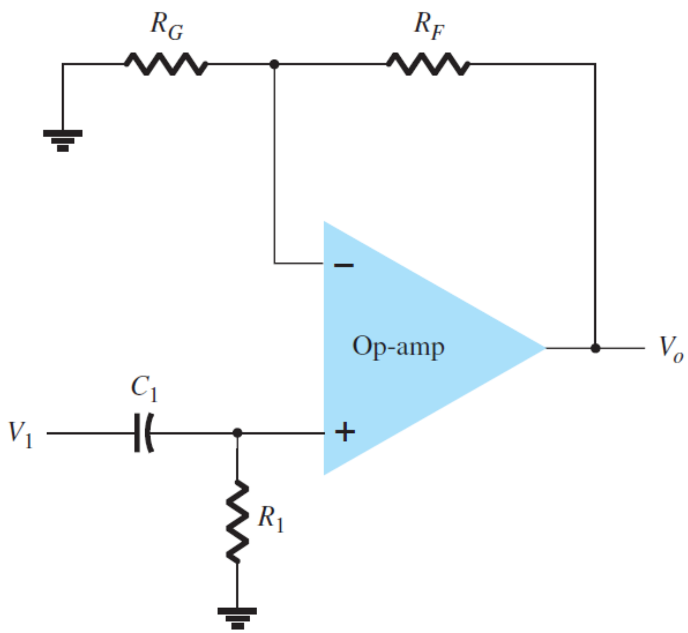
RC High-pass filter

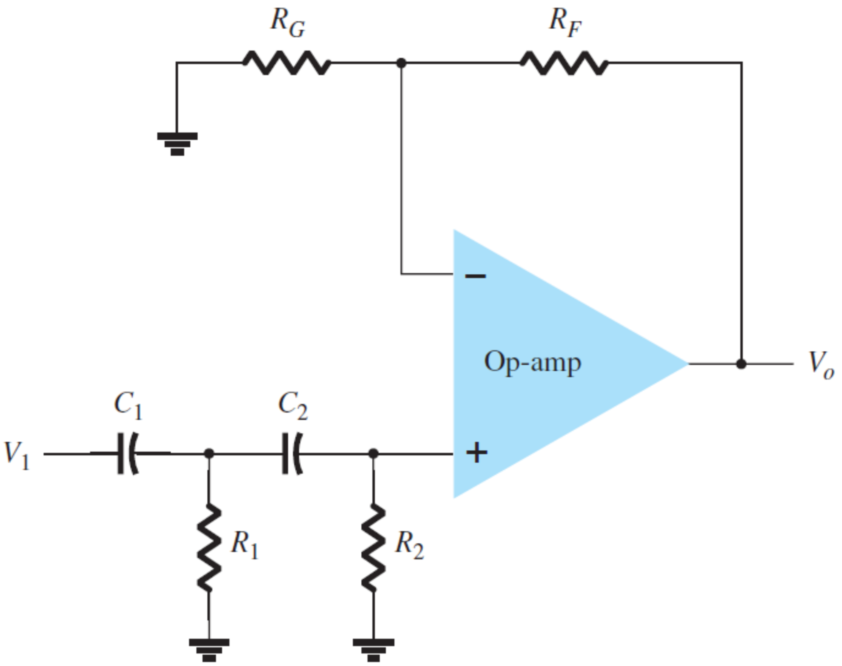


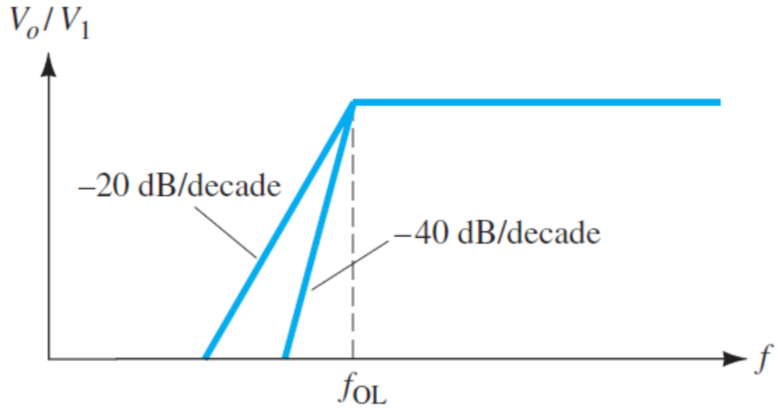
$$A_v = \left(1 + \frac{R_F}{R_G}\right)$$

$$f_{OH} = \frac{1}{2\,\pi R_1 C_1}$$

Active Filters – High-Pass Filters

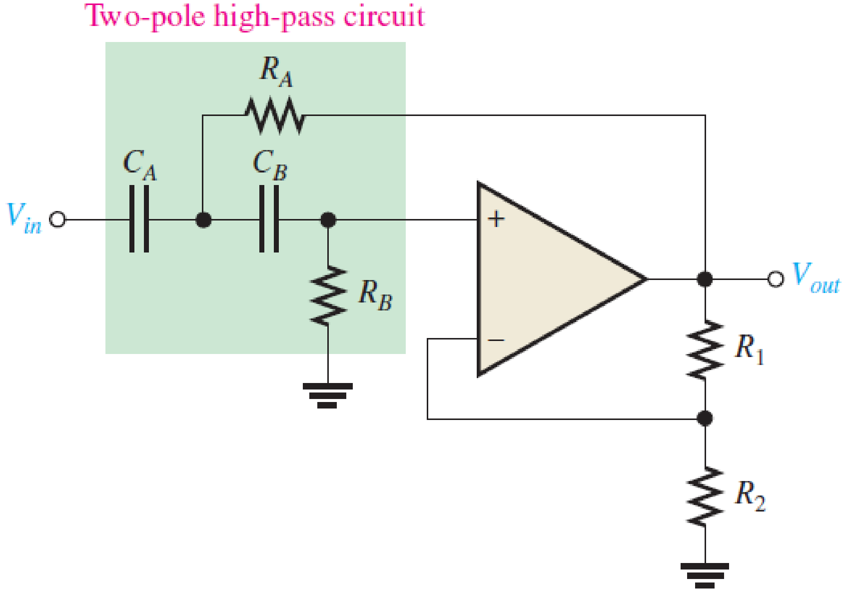






Active Filters – High-Pass Filters

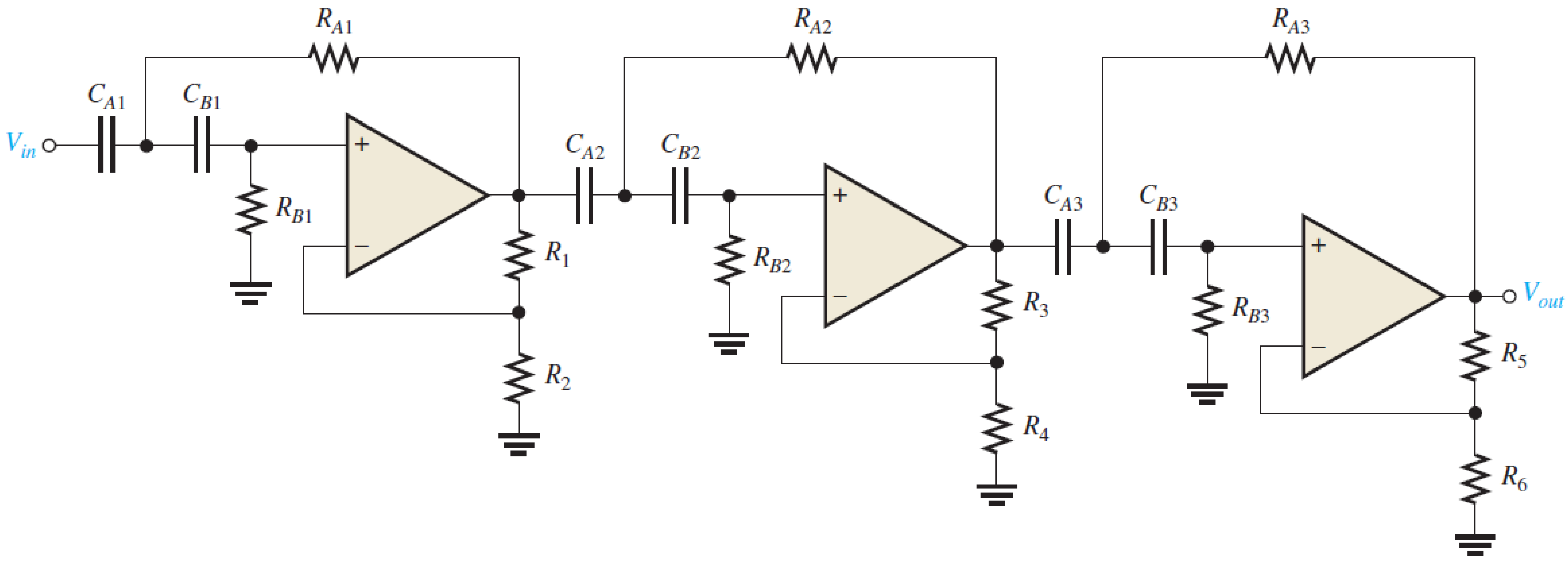
The Sallen-Key High-Pass Filter



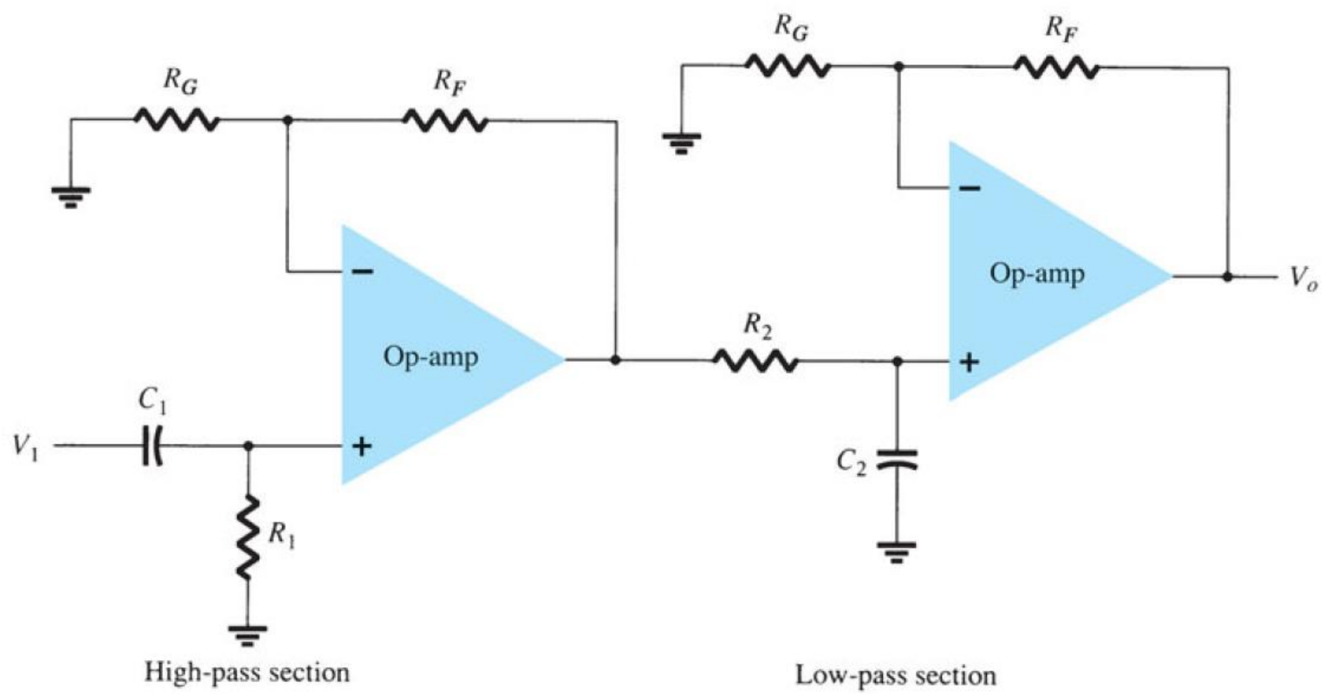
- Components R_A, C_A, R_B and C_B form the two-pole frequency-selective circuit
- Note that the positions of the resistors and capacitors in the frequency-selective circuit are opposite to those in the low-pass configuration
- As with the other filters, the response characteristic can be optimized by proper selection of the feedback resistors, R_1 and R_2.

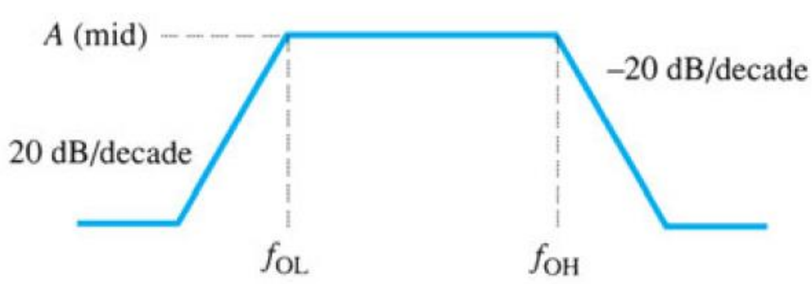
Active Filters – High-Pass Filters

Cascading High-Pass Filters

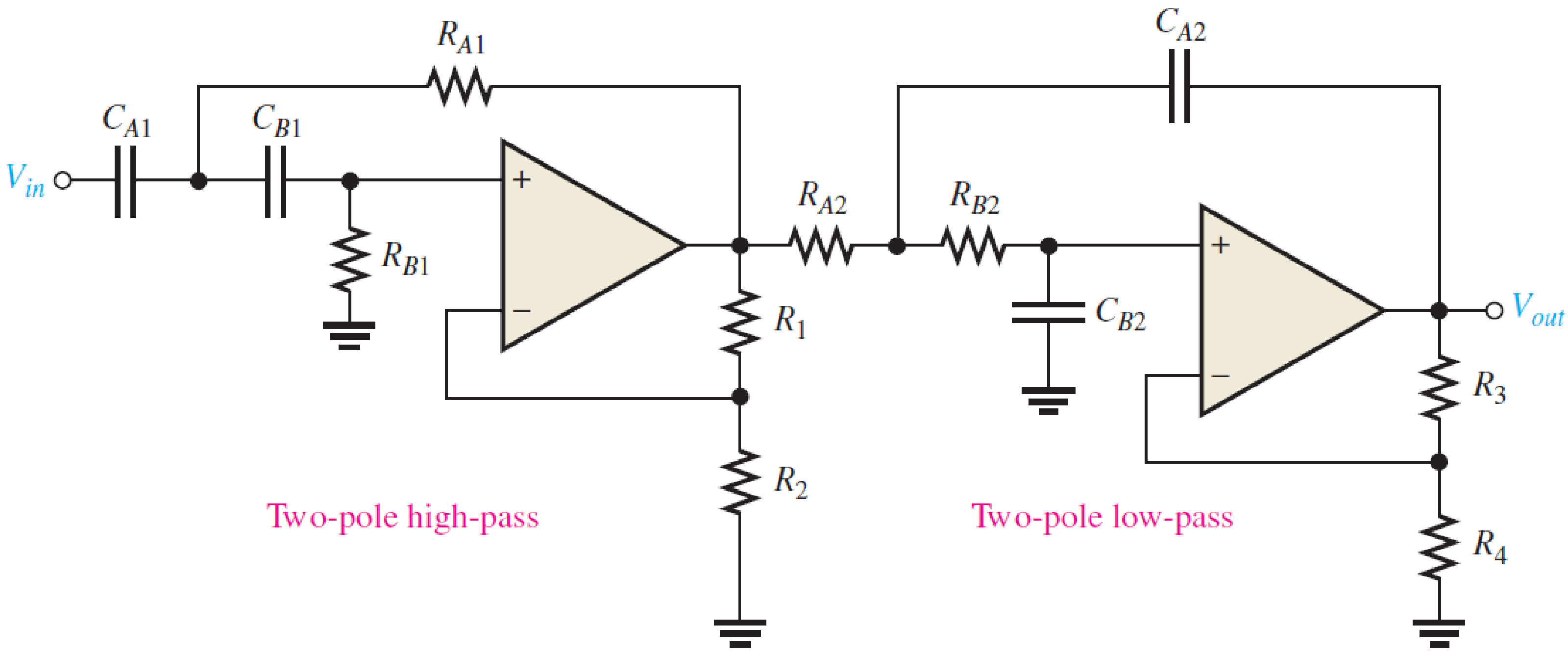


Active Filters – Band-Pass Filters





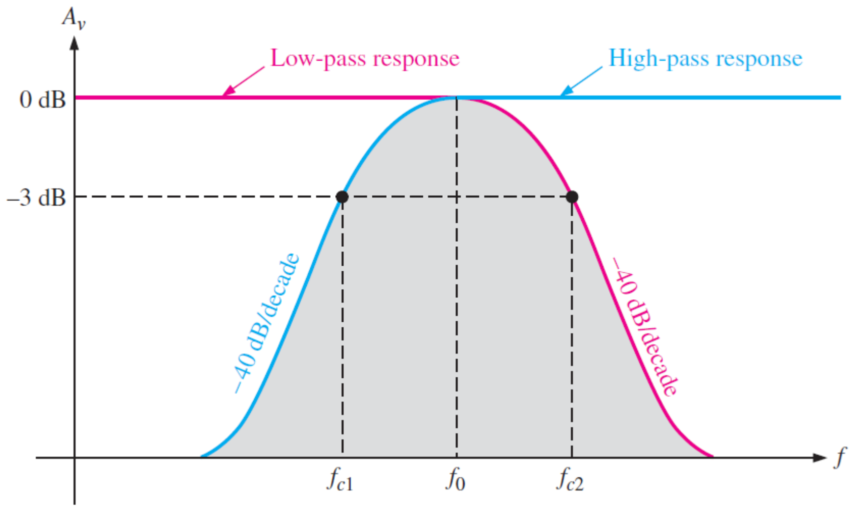
Active Filters – Band-Pass Filters



$$f_{c1} = \frac{1}{2\pi\sqrt{R_{A1}R_{B1}C_{A1}C_{B1}}}$$

$$f_{c2} = \frac{1}{2\pi\sqrt{R_{A2}R_{B2}C_{A2}C_{B2}}}$$

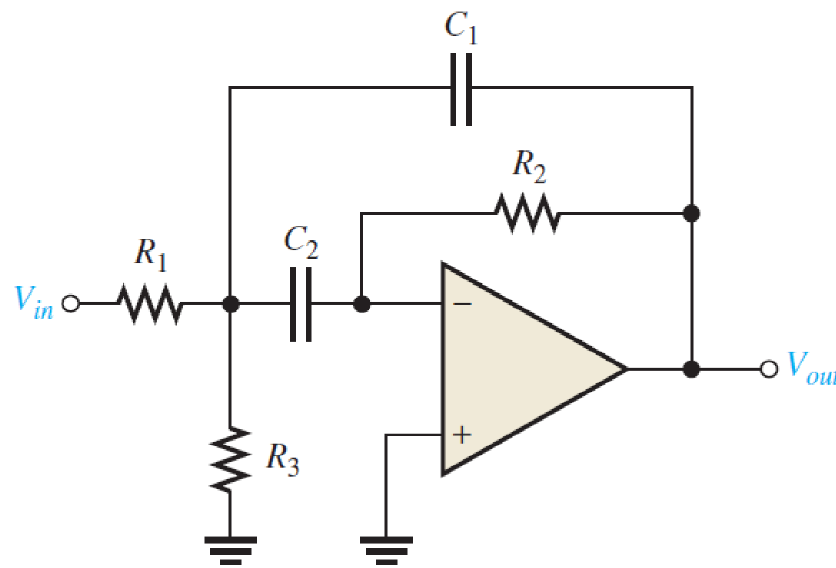
$$f_0 = \sqrt{f_{c1}f_{c2}}$$



Active Filters – Band-Pass Filters

Multiple-Feedback Band-Pass Filter

- The two feedback paths are through R_2 and C_1
- R_1 and C_1 provide low-pass response
- R_2 and C_2 provide high-pass response
- Maximum gain, A_0, occurs at the center frequency
- Q values of less than 10 are typical in this type of filter.



R_1 and R_3 appear in parallel as viewed from the C_1 feedback path (with the V_{in} source replaced by a short).

$$f_0 = \frac{1}{2\pi\sqrt{(R_1 \| R_3)R_2C_1C_2}}$$

Making $C_1 = C_2 = C$ yields

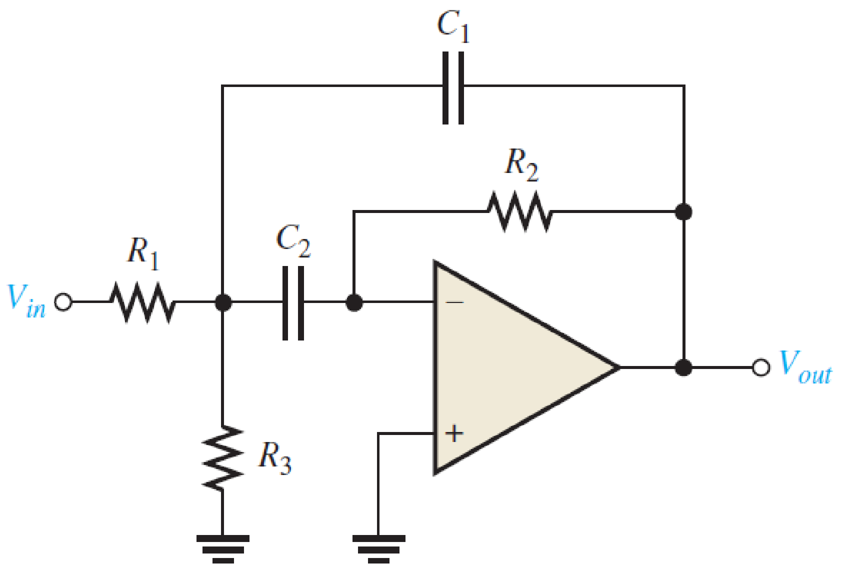
$$f_0 = \frac{1}{2\pi\sqrt{(R_1 \| R_3)R_2C^2}} = \frac{1}{2\pi C\sqrt{(R_1 \| R_3)R_2}}$$

$$= \frac{1}{2\pi C}\sqrt{\frac{1}{R_2(R_1 \| R_3)}} = \frac{1}{2\pi C}\sqrt{\left(\frac{1}{R_2}\right)\left(\frac{1}{R_1R_3/R_1 + R_3}\right)}$$

$$\mathbf{f_0 = \frac{1}{2\pi C}\sqrt{\frac{R_1 + R_3}{R_1R_2R_3}}}$$

Active Filters – Band-Pass Filters

$$f_0 = \frac{1}{2\pi C}\sqrt{\frac{R_1 + R_3}{R_1 R_2 R_3}}$$



- A value for the capacitors is chosen and then the three resistor values are calculated to achieve the desired values for f_0, BW, and A_0
- $Q = f_0/BW$
- Resistor values can be found using the following formulas (stated without derivation):

$$R_1 = \frac{Q}{2\pi f_0 C A_0}$$

$$R_2 = \frac{Q}{\pi f_0 C}$$

$$R_3 = \frac{Q}{2\pi f_0 C(2Q^2 - A_0)}$$

$$Q = 2\pi f_0 A_0 C R_1$$

$$Q = \pi f_0 C R_2$$

$$2\pi f_0 A_0 C R_1 = \pi f_0 C R_2$$

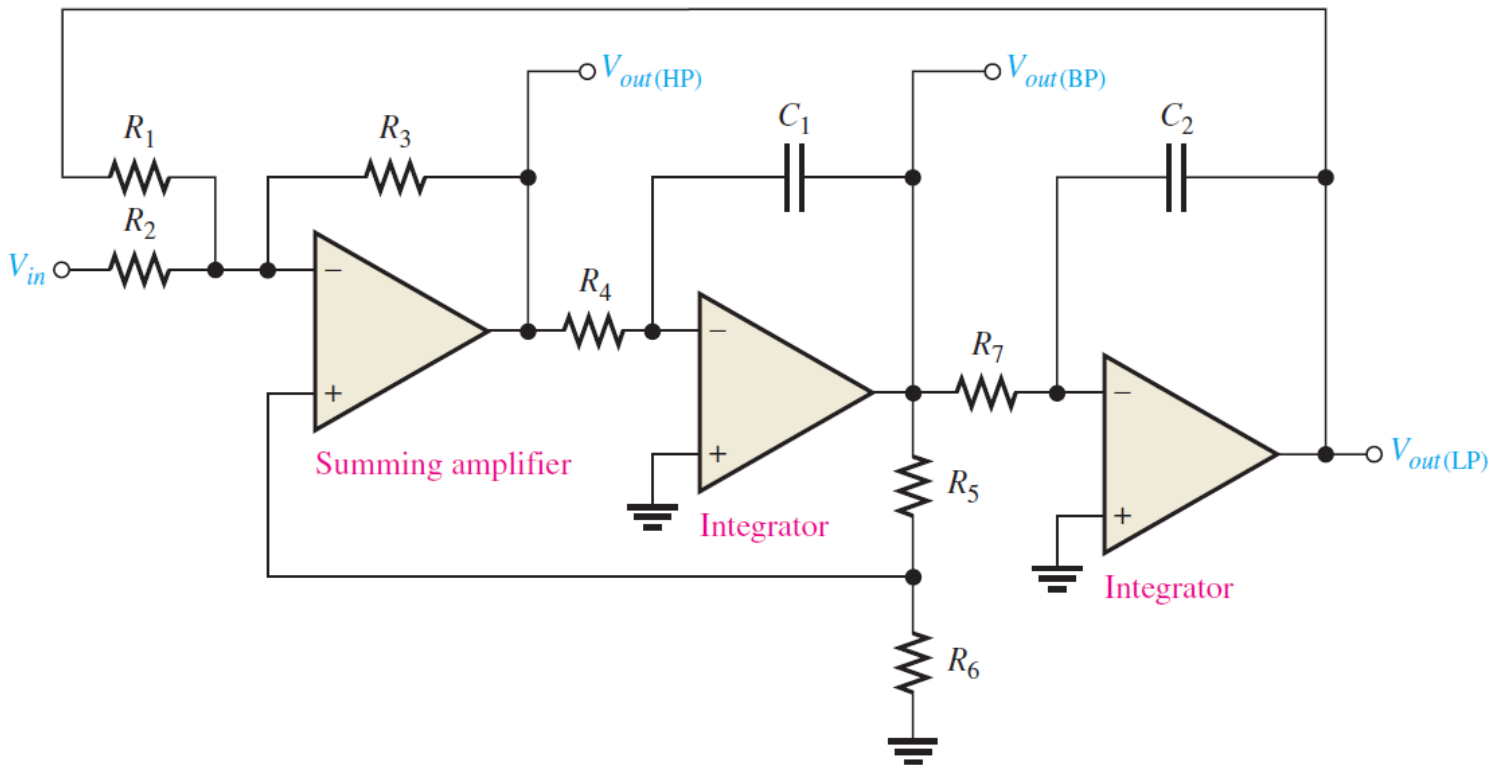
$$2A_0 R_1 = R_2$$

$$A_0 = \frac{R_2}{2R_1}$$

- For denominator of the expression above to be positive, $A_0 < 2Q^2$
- => a limitation on gain.

Active Filters

State-Variable Filter



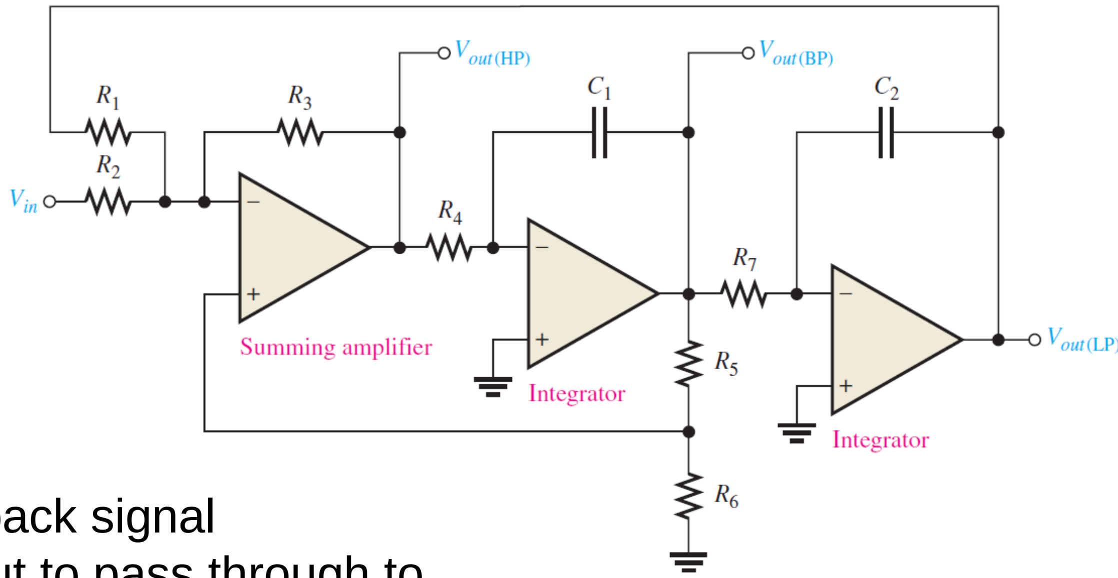
Consists of a summing amplifier and two op-amp integrators
Integrators act as single-pole low-pass filters combined in cascade to form a second-order filter
Although used primarily as a band-pass (BP) filter, it also provides low-pass (LP) and high-pass (HP) outputs

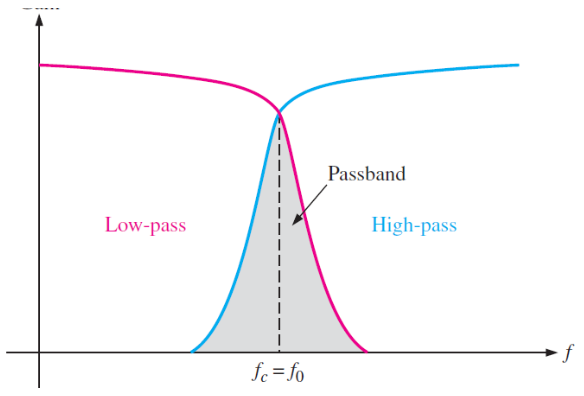
Active Filters – Band-Pass Filters

State-Variable Filter

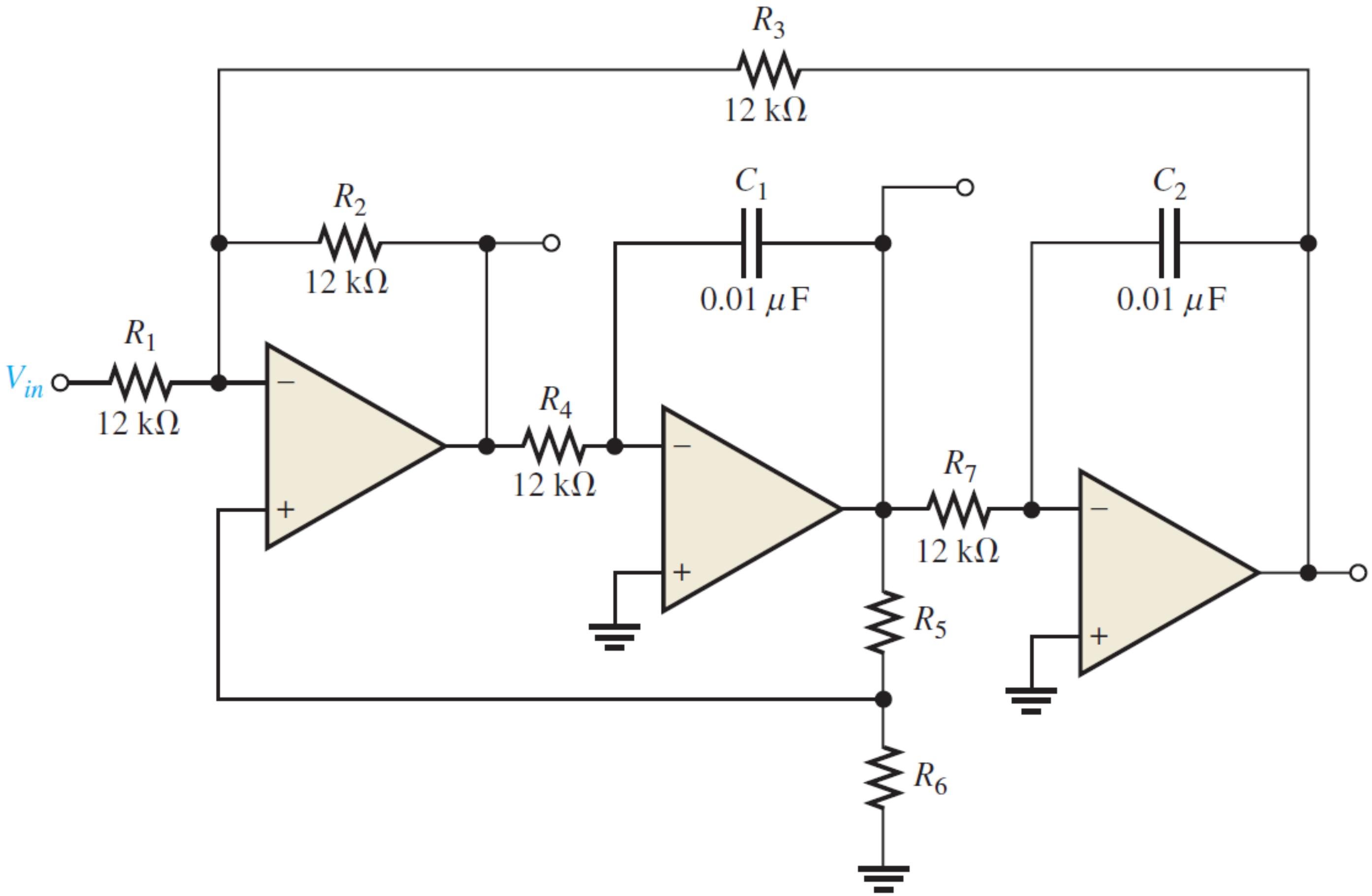
- At input frequencies below f_c, input signal passes through the summing amplifier and integrators and fed back out of phase
- Thus, the feedback signal and input signal cancel for all frequencies below f_c.
- At higher frequencies, feedback signal diminishes, allowing the input to pass through to the band-pass output
- As a result, BP output peaks sharply at f_c
- Stable Qs up to 100 can be obtained
- Q is set by the feedback resistors R5 and R6 according to equation:

$$Q = \frac{1}{3}\left(\frac{R_5}{R_6} + 1\right)$$



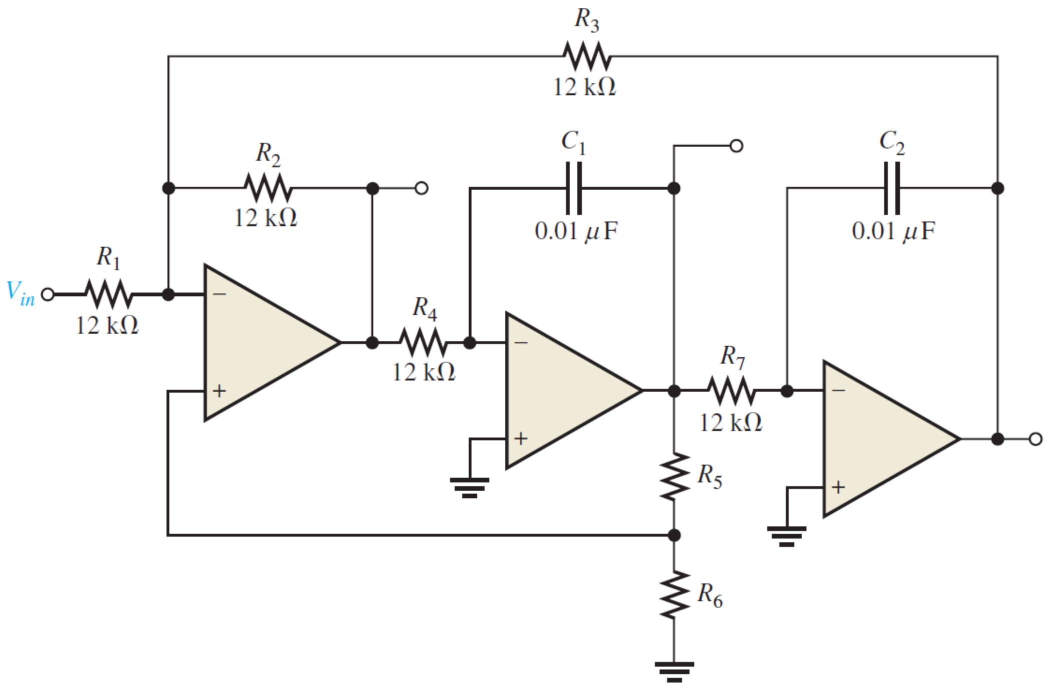


Active Filters – Band-Stop Filters

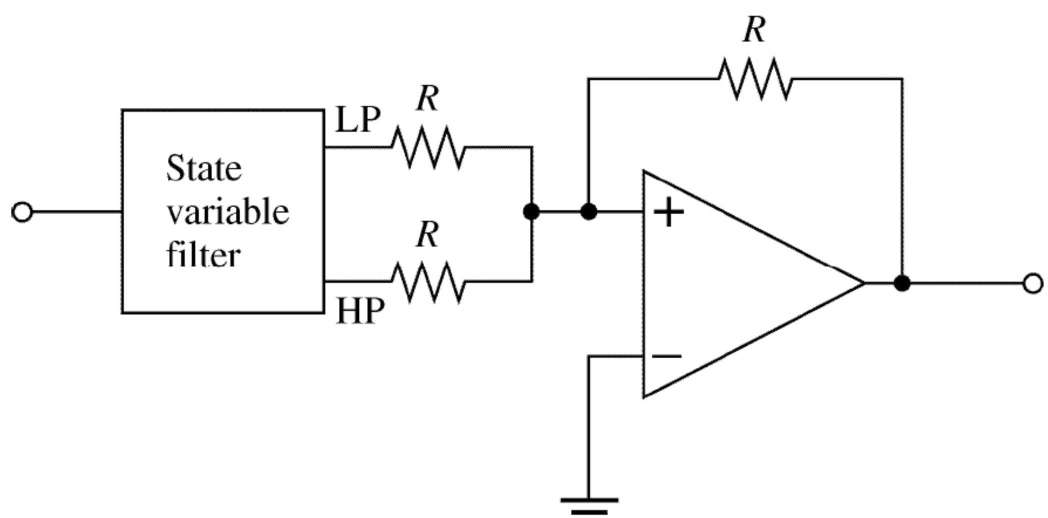


Show how to make a notch (band-stop) filter using the circuit

Active Filters – Band-Stop Filters



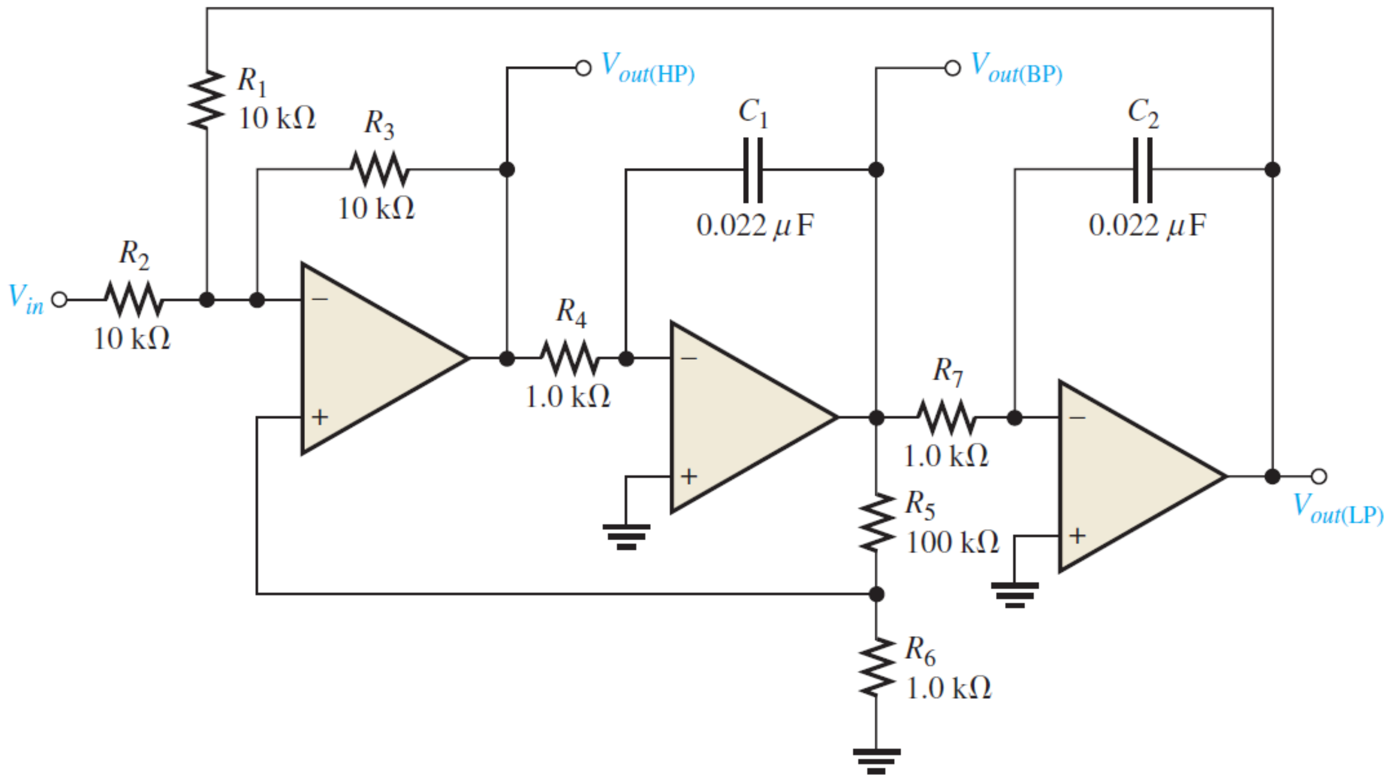
Show how to make a notch (band-stop) filter using the circuit



Active Filters – Band-Pass Filters

State-Variable Filter

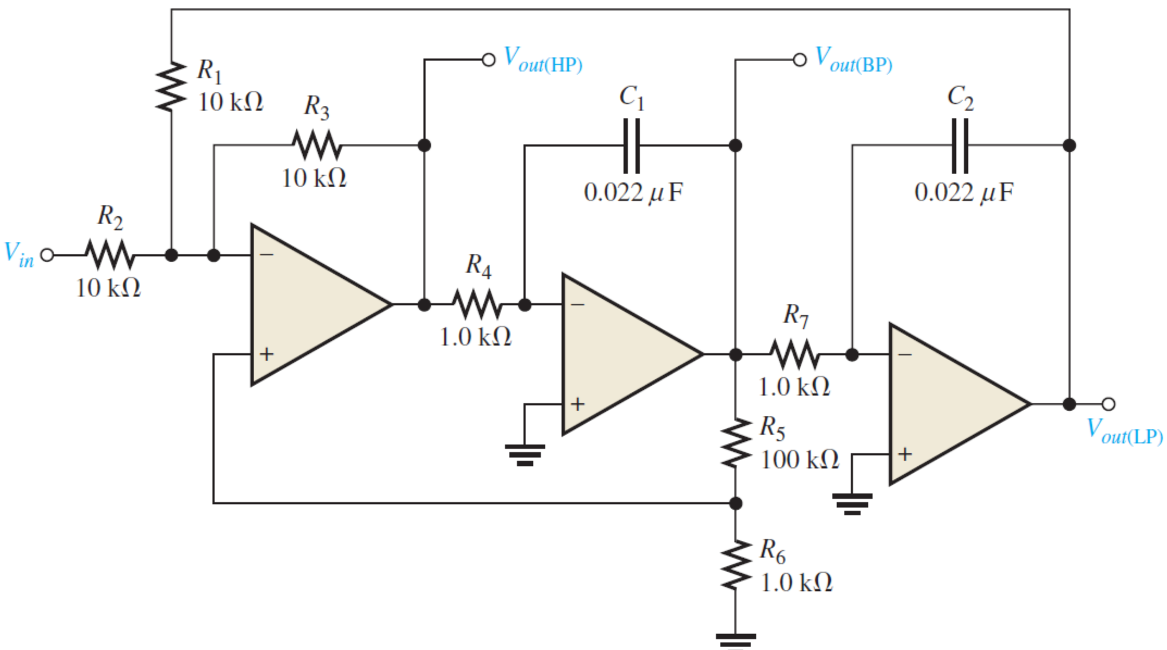
Determine the center frequency, Q, and BW for the passband of the filter



Active Filters – Band-Pass Filters

State-Variable Filter

Determine the center frequency, Q, and BW for the passband of the filter



For each integrator

$$f_c = \frac{1}{2\pi R_4 C_1} = \frac{1}{2\pi R_7 C_2} f_c$$

$$= \frac{1}{2\pi(1.0k\Omega)(0.22\ \mu F)} = 7.23\ kHz$$

$$f_0 = f_c = 7.23\ kHz$$

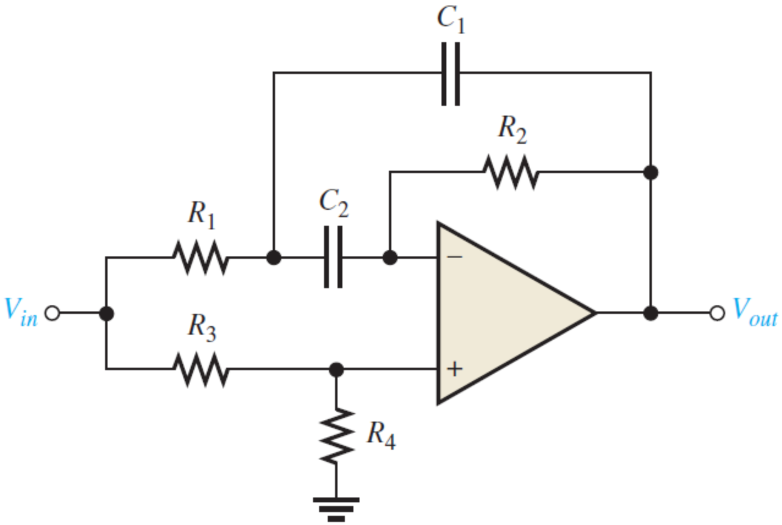
$$Q = \frac{1}{3}\left(\frac{R_5}{R_6} + 1\right) = \frac{1}{3}\left(\frac{100\ k\Omega}{1.0\ k\Omega} + 1\right)$$

$$= 33.7$$

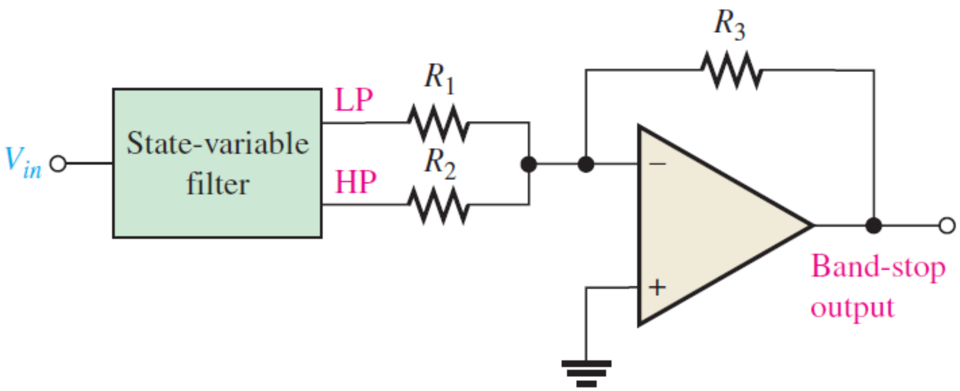
$$BW = \frac{f_0}{Q} = \frac{7.23\ kHz}{33.7} = 215\ Hz$$

Active Filters – Band-Stop Filters

Multiple-Feedback Band-Stop Filter



State-Variable Band-Stop Filter

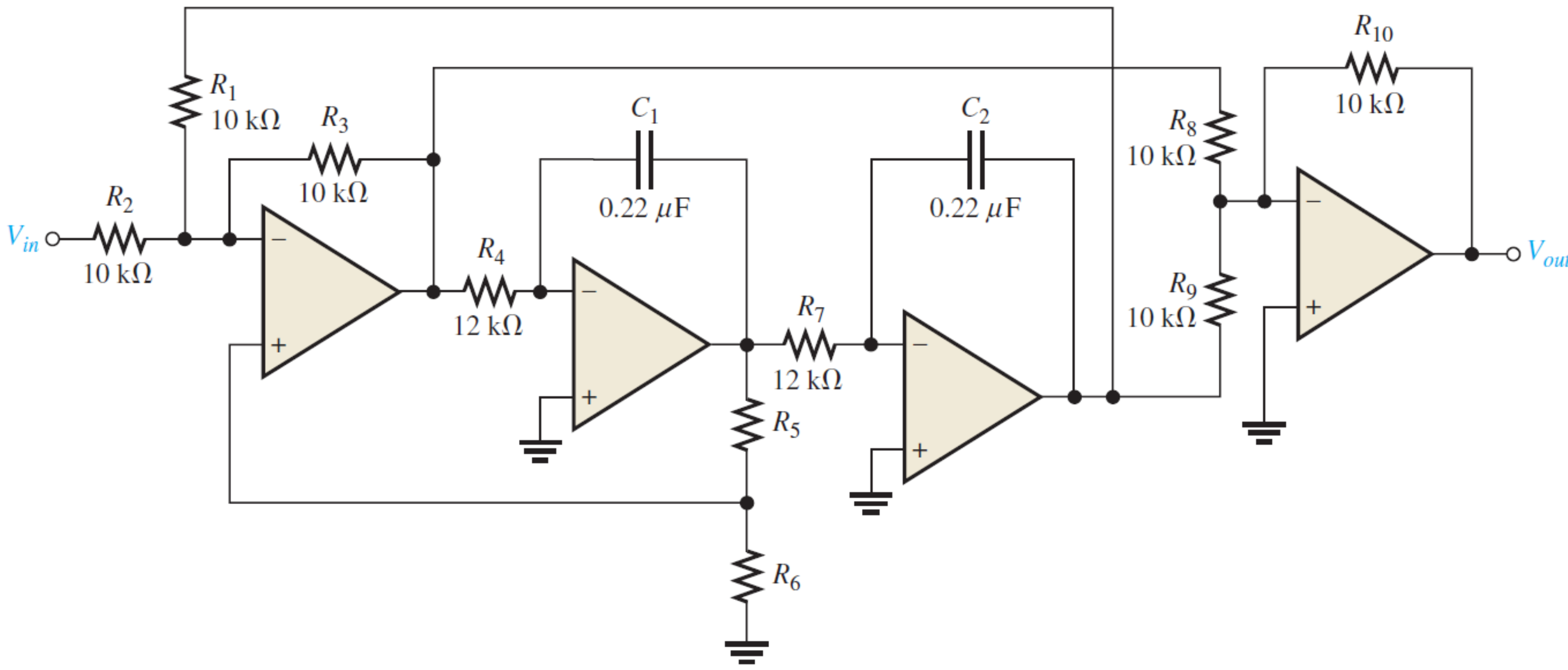


One important application of this filter is minimizing the 50 Hz “hum” in audio systems by setting the center frequency to 50 Hz

Active Filters – Band-Stop Filters

State-Variable Band-Stop Filter

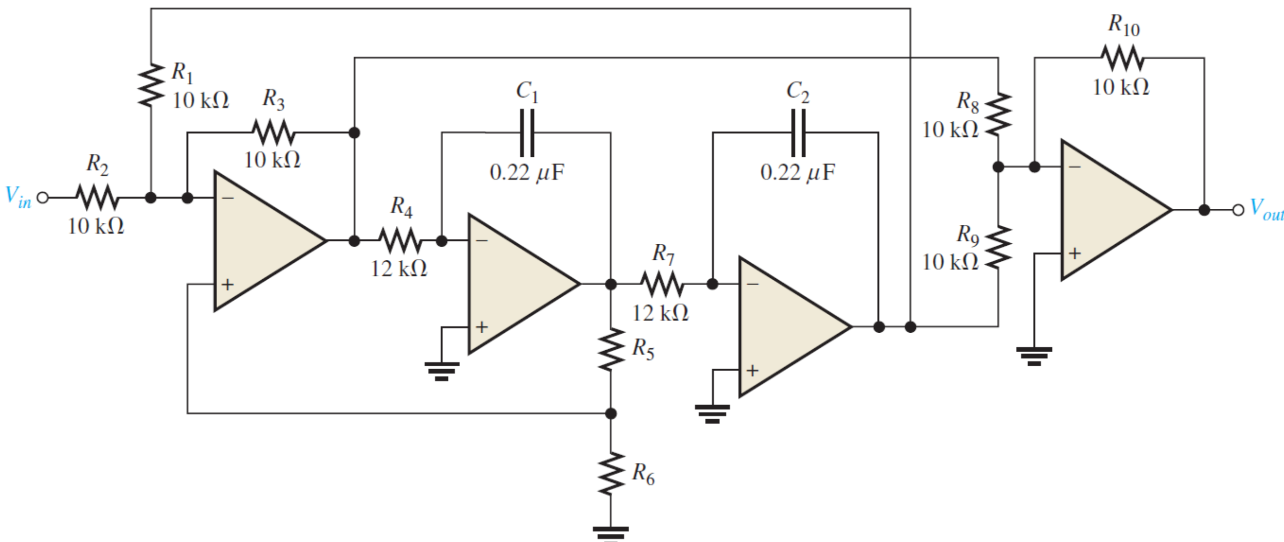
Verify that the band-stop filter in the figüre has a center frequency of 60 Hz, and optimize the filter for a *Q* of 10



Active Filters – Band-Stop Filters

State-Variable Band-Stop Filter

Verify that the band-stop filter in figure has a center frequency of 60 Hz, and optimize the filter for a *Q* of 10



For each integrator

$$f_c = f_0 = \frac{1}{2\pi R_4 C_1} = \frac{1}{2\pi R_7 C_2} f_c$$

$$= \frac{1}{2\pi(12k\Omega)(0.22\ \mu F)} = 60\ Hz$$

$$Q = \frac{1}{3}\left(\frac{R_5}{R_6} + 1\right)$$

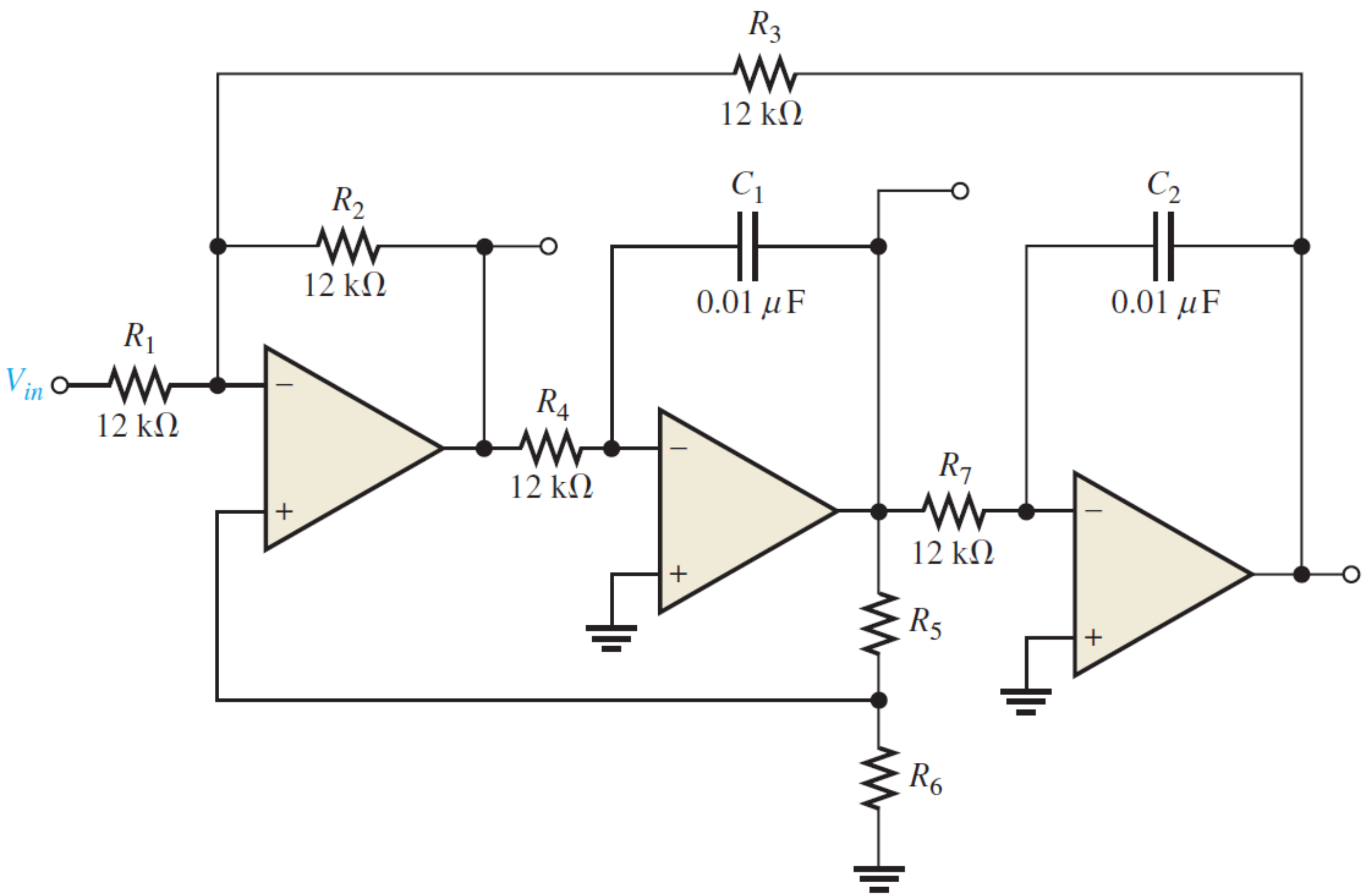
$$R_5 = (3Q - 1)R_6$$

Choose $R_6 = 3.3\ k\Omega$

$$R_5 = [3(10) - 1]3.3k\Omega$$

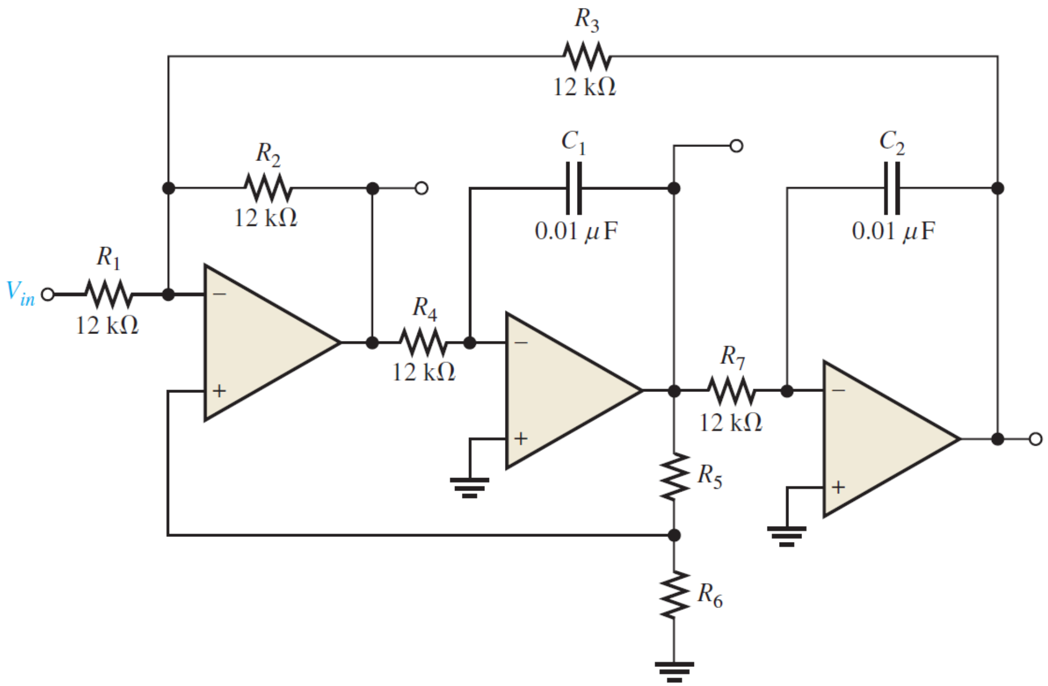
$$= 95.7\ k\ \Omega$$

Active Filters – Band-Stop Filters



Optimize the state-variable filter for Q=50.
What bandwidth is achieved?

Active Filters – Band-Stop Filters



$$Q = \frac{1}{3}\left(\frac{R_5}{R_6} + 1\right)$$

Select $R_6 = \mathbf{10\ k\Omega}$.

$$Q = \frac{R_5}{3R_6} + \frac{1}{3} = \frac{R_5 + R_6}{3R_6}$$

$$3R_6Q = R_5 + R_6$$

$$R_5 = 3R_6Q - R_6$$

$$= 3(10\ k\Omega)(50) - 10\ k\Omega$$

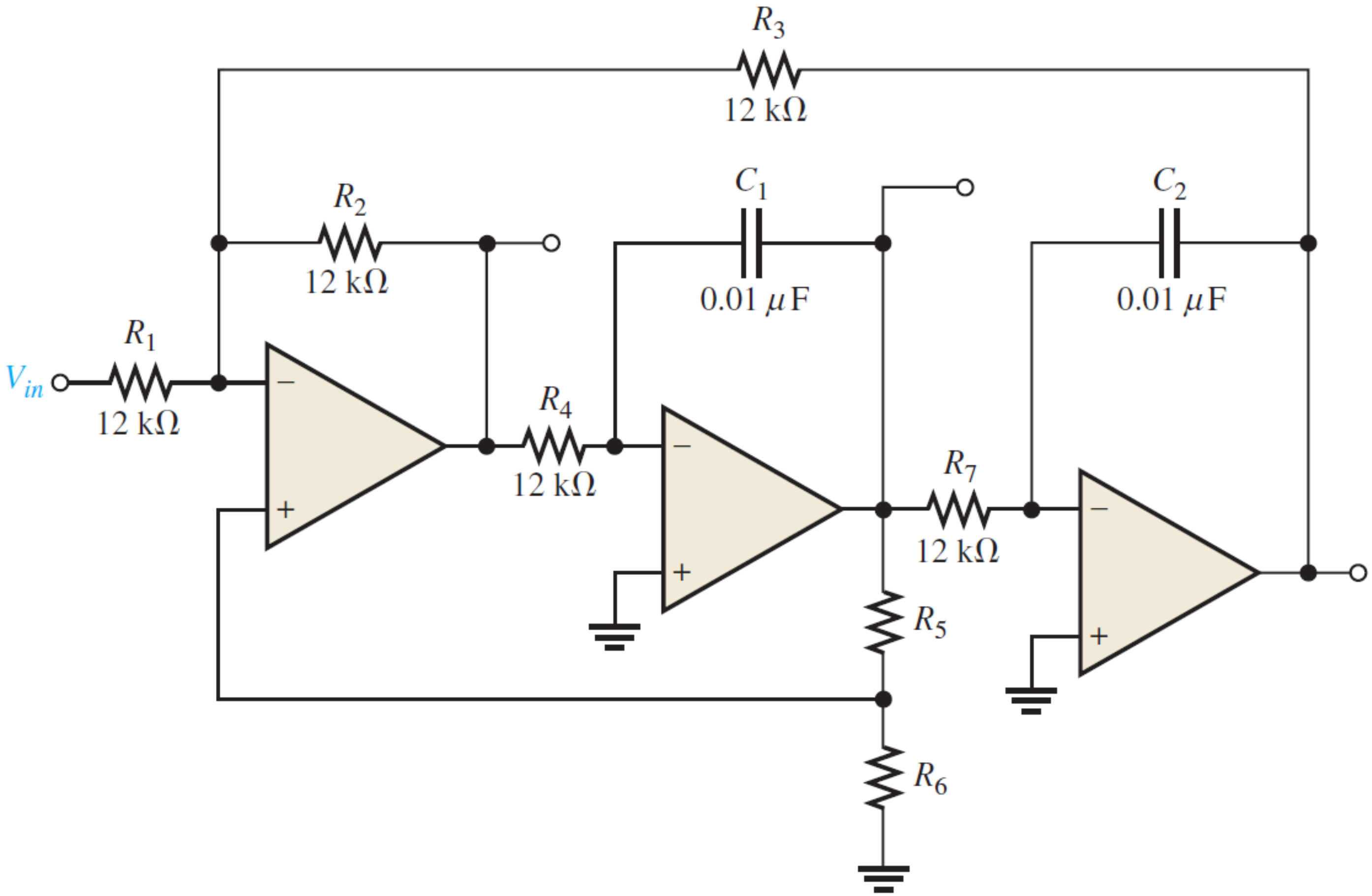
$$= 1500\ k\Omega - 10\ k\Omega = \mathbf{1490\ k\Omega}$$

$$f_0 = \frac{1}{2\pi(12\ k\Omega)(0.01\ \mu F)} = 1.33\ kHz$$

$$BW = \frac{f_0}{Q} = \frac{1.33\ kHz}{50} = \mathbf{26.6\ Hz}$$

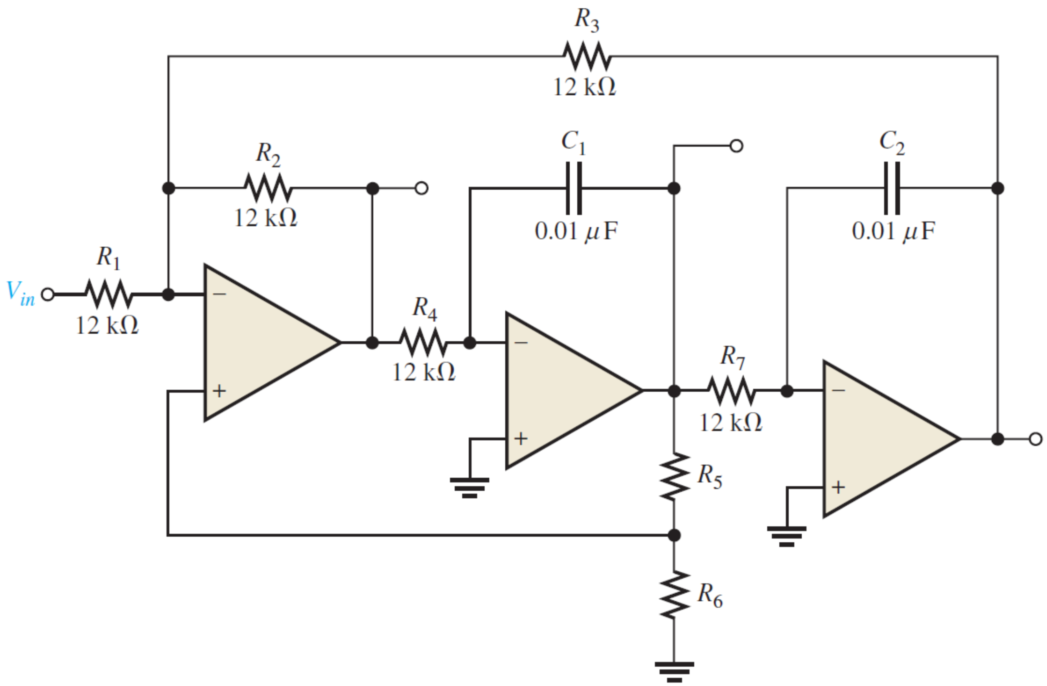
Optimize the state-variable filter for Q=50.
What bandwidth is achieved?

Active Filters – Band-Stop Filters



Modify the band-stop filter for a center frequency of 120 Hz

Active Filters – Band-Stop Filters



Modify the band-stop filter for a center frequency of 120 Hz

$$f_0 = f_c = \frac{1}{2\pi RC}$$

Let C remain 0.01 μF.

$$R = \frac{1}{2\pi f_0 C} = \frac{1}{2\pi(120\text{ Hz})(0.01\ \mu\text{F})} = \mathbf{133\ k\Omega}$$

Change R in the integrators from 12 kΩ to 133 kΩ.

Active Filters – Band-Stop Filters

State-Variable Band-Stop Filter

Find typical resistor and capacitor values for a center frequency of 50 Hz.

Standard Resistor Values (±5%)

1.0	10	100	1.0K	10K	100K	1.0M
1.1	11	110	1.1K	11K	110K	1.1M
1.2	12	120	1.2K	12K	120K	1.2M
1.3	13	130	1.3K	13K	130K	1.3M
1.5	15	150	1.5K	15K	150K	1.5M
1.6	16	160	1.6K	16K	160K	1.6M
1.8	18	180	1.8K	18K	180K	1.8M
2.0	20	200	2.0K	20K	200K	2.0M
2.2	22	220	2.2K	22K	220K	2.2M
2.4	24	240	2.4K	24K	240K	2.4M
2.7	27	270	2.7K	27K	270K	2.7M
3.0	30	300	3.0K	30K	300K	3.0M
3.3	33	330	3.3K	33K	330K	3.3M
3.6	36	360	3.6K	36K	360K	3.6M
3.9	39	390	3.9K	39K	390K	3.9M
4.3	43	430	4.3K	43K	430K	4.3M
4.7	47	470	4.7K	47K	470K	4.7M
5.1	51	510	5.1K	51K	510K	5.1M
5.6	56	560	5.6K	56K	560K	5.6M
6.2	62	620	6.2K	62K	620K	6.2M
6.8	68	680	6.8K	68K	680K	6.8M
7.5	75	750	7.5K	75K	750K	7.5M
8.2	82	820	8.2K	82K	820K	8.2M
9.1	91	910	9.1K	91K	910K	9.1M

Electrolytic Capacitor Values Chart

0.1 µF	68 µF	480 µF
0.15 µF	72 µF	500 µF
0.22 µF	75 µF	510 µF
0.33 µF	82 µF	520 µF
0.47 µF	88 µF	540 µF